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IMPERFECTION SENSITIVITY ANALYSIS OF COMPOSITE CYLINDRICAL SHELLS USING KOITER'S METHOD

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Abstract

The novel methodology for imperfection sensitivity analysis, presented in Barbero et al.[1] is here applied for the evaluation of limit load of composite cylindrical shells. Koiter's perturbation method is used to calculate the imperfection paths emanating from mode interaction bifurcations and the Monte Carlo method is used to test a large number of modes and all possible interactions among them. The computational cost is low because of the efficiency of Koiter's method. The demands of Koiter's method for accurate evaluations of higher order derivatives of the potential energy are met by a mixed, corotational element.

keywords: Koiter's asymptotic approach; Composite Cylinder; Corotational kinematics; Monte Carlo; Imperfection sensitivity.

1 Introduction

The evaluation of the structural performance of cylinders in compression is a classical technical problem and applications to space launchers, off-shore platforms, and so on [2]. The design of cylindrical sandwich shells structures is generally dominated by interactive buckling. It is well known that experimental limit load load are much lower than predicted by buckling loads. This is explained by the presence of imperfections, of different nature, in real cylinders and interactive buckling that affect the load carrying capacity. It is clear that the presence of imperfections greatly influence the structural behaviour of the cylinder under compression. Therefore it is necessary to evaluate the imperfection sensitivity of this type of structures. Imperfection sensitivity analysis requires the identification of a large number of buckling modes and their interaction. Because of the large number of possible modes and the uncertainly about

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which ones produce the most dangerous interaction among them, such analysis is prohibitively time consuming. Continuation methods based on Riks method are often used. In spite of the simplicity of its numerical implementation, which requires only an approximation of the tangent stiffness matrix, Riks method suffers in the case of multiple bifurcations, requiring ad-hoc branch switch algorithms. A valid alternative for these cases is the explicit dynamic method but such method takes long simulation time because the time increment may need to be very small. Moreover, both Riks and explicit dynamics methods suffer from another drawback: the analysis must be fully repeated for each imperfection, making impracticable when combined with Monte Carlo simulation. Therefore, the aim of this work is to apply a robust and efficient methodology [1] to study the imperfection sensitivity of cylindrical sandwich shells under axial compression. The proposed methodology does not require a priori knowledge of the shape and magnitude of imperfections and does not rely on lengthy continuation analysis. Instead, it uses Koiter's perturbation approach [3, 4] to calculate the bifurcation load, post-buckling path, and interaction between modes to detect bifurcations on the post-buckling path of individual modes, as well as the paths emanating from those bifurcations. The approach is based on a fourth-order energy expansion, thus requiring a geometrically coherent structural model for reliability of the analysis [5]. The corotational approach fulfills this requirement, allowing also complete reuse of a linear model and its corresponding finite element for the geometrically nonlinear analysis. A Hellinger-Reissner formulation is used to avoid extrapolation looking. The recent 3D plate finite element [6, 7], provides evaluation of linear elastic response and rotation fields. Therefore, it is very suitable to be used along with a corotational formulation to perform geometrically nonlinear analysis [5]. Using Monte Carlo simulation, for a random sequence of imperfections, each one obtained as a linear combination of buckling modes, the equilibrium paths for the imperfect structures are recovered. Then, the worst imperfection is detected and the corresponding limit load is obtained. The proposed methodology allows us to run thousand of analysis in the time of a single run of Riks' or explicit dynamic analysis [1].

2 Koiter's asymptotic analysis

Asymptotic approach is essentially the implementation of Koiter's nonlinear elastic stability approach [8] into the finite element method (FEM) [3, 4]. The solution process is based on an expansion of the potential energy in terms of load factor λ and modal amplitudes ξ_i. It can be summarized as follows:

1. The *fundamental path* is obtained as a linear extrapolation

$$\boldsymbol{u}^f[\lambda] = \boldsymbol{u}_0 + \lambda \hat{\boldsymbol{u}} \tag{1a}$$

   where $\boldsymbol{u}_0$ is an initial displacement, possibly null, and $\boldsymbol{u} = \lambda \hat{\boldsymbol{u}}$ is the vector of kinematic parameters, i.e., the space of degrees of freedom (dof) of the structure and $\hat{\boldsymbol{u}} = d\boldsymbol{u}/d\lambda$ is obtained as the solution of the linear algebraic equation

$$\boldsymbol{K}_0 \, \hat{\boldsymbol{u}} = \hat{\boldsymbol{p}} \tag{1b}$$

where $\hat{\boldsymbol{p}}$ is the reference load and $\boldsymbol{K}_0 = \boldsymbol{K}[\boldsymbol{u}_0]$ is the stiffness matrix, which contains the coefficients of the quadratic terms of the energy Φ''.

2. A cluster of *buckling loads* λ_i, $i = 1 \cdots m$, and associated *buckling modes* $\dot{\boldsymbol{v}}_i$ are obtained along $\boldsymbol{u}^f[\lambda]$ by the critical condition

$$\boldsymbol{K}[\lambda_i] \, \dot{\boldsymbol{v}}_i = \mathbf{0} \quad , \quad \boldsymbol{K}[\lambda] = \boldsymbol{K}[\boldsymbol{u}_0 + \lambda \hat{\boldsymbol{u}}] \tag{1c}$$

the eigenvalue problem is defined as fully nonlinear, to correctly recover the post-critical behavior. The nonlinearity is introduced by updating the configuration along the fundamental path. Note that the size m of the subspace of buckling modes needed for the analysis is of orders of magnitude smaller than the number of dof used to discretize the structure.

We will denote with $\mathcal{V} = \{\dot{\boldsymbol{v}} = \sum_{i=1}^{m} \xi_i \dot{\boldsymbol{v}}_i\}$ the subspace spanned by the buckling modes $\dot{\boldsymbol{v}}_i$ and $\mathcal{W} = \{\boldsymbol{w} : \boldsymbol{w} \perp \dot{\boldsymbol{v}}_i \, , \; i = 1 \cdots m\}$ its orthogonal complement according the orthogonality condition

$$\boldsymbol{w} \perp \dot{\boldsymbol{v}}_i \;\Leftrightarrow\; \Phi_b''' \hat{u} \dot{v}_i w = 0 \tag{1d}$$

where $\hat{u} = \mathcal{L}\hat{\boldsymbol{u}}$, $\dot{v}_i = \mathcal{L}\dot{\boldsymbol{v}}_i$, $w = \mathcal{L}\boldsymbol{w}$ and $\mathcal{L}$ the linear operator of FEM interpolation. We will also denote with λ_b an appropriate reference value for the cluster, e.g. the smallest of λ_i or their mean value, and with a suffix "b" quantities evaluated in correspondence to $\boldsymbol{u}_b = \boldsymbol{u}^f[\lambda_b]$.

3. Denoting $\xi_0 = (\lambda - \lambda_b)$ and $\dot{\boldsymbol{v}}_0 = \hat{\boldsymbol{u}}$, the asymptotic approximation for any equilibrium path is approximated by an expansion in terms of mode amplitudes ξ_j as follows

$$\boldsymbol{u}[\lambda, \xi_k] = \boldsymbol{u}_b + \sum_{i=0}^{m} \xi_i \dot{\boldsymbol{v}}_i + \frac{1}{2} \sum_{i,j=0}^{m} \xi_i \xi_j \boldsymbol{w}_{ij} \tag{1e}$$

where $\boldsymbol{w}_{ij} \in \mathcal{W}$ are quadratic corrections introduced to satisfy the projection of the equilibrium equation into $\mathcal{W}$, obtained by the linear *orthogonal equations*

$$\delta \boldsymbol{w}^T (\boldsymbol{K}_b \boldsymbol{w}_{ij} + \boldsymbol{p}_{ij}) = 0 \; , \;\; \forall \boldsymbol{w} \in \mathcal{W} \tag{1f}$$

where $\boldsymbol{K}_b = \boldsymbol{K}[\boldsymbol{u}^f[\lambda_b]]$ and vectors $\boldsymbol{p}_{ij}$ are defined as a function of modes $\dot{\boldsymbol{v}}_i$ and, $i = 0 \cdots m$ by the energy equivalence $\delta \boldsymbol{w}^T \boldsymbol{p}_{ij} = \Phi_b''' \delta w \, \dot{v}_j \dot{v}_j$.

4. The following energy terms are computed for $i, j = 0 \cdots m$, $k = 1 \cdots m$:

$$\begin{aligned}
\mathcal{A}_{ijk} &= \Phi_b''' \dot{v}_i \dot{v}_j \dot{v}_k \\
\mathcal{B}_{ijhk} &= \Phi_b'''' \dot{v}_i \dot{v}_j \dot{v}_h \dot{v}_k - \Phi_b'' (w_{ij} w_{hk} + w_{ih} w_{jk} + w_{ik} w_{jh}) \\
\mathcal{C}_{ik} &= \Phi_b'' w_{00} w_{ik} \\
\mu_k[\lambda] &= \frac{1}{2} \lambda_b (\lambda - \frac{1}{2} \lambda_b) \Phi_b''' \hat{u}^2 \dot{v}_k + \frac{1}{6} \lambda_b^2 (\lambda_b - 3\lambda) \Phi_b'''' \hat{u}^3 \dot{v}_k
\end{aligned} \tag{1g}$$

where the *implicit imperfection factors* μ_k are defined by the 4th order expansion of the unbalanced work on the fundamental path (i.e. $\mu_k[\lambda] = (\lambda\hat{p} - \Phi'[\lambda\hat{u}])\dot{v}_k$).

5. The equilibrium path is obtained by projecting equilibrium equation on $\mathcal{V}$. According to Eqs, (1a)–(1g), we have

$$\begin{aligned}&\frac{1}{2}\sum_{i,j=0}^{m}\xi_i\xi_j\mathcal{A}_{ijk} + \frac{1}{6}\sum_{i,j,h=0}^{m}\xi_i\xi_j\xi_h\mathcal{B}_{ijhk} + \mu_k[\lambda]\\ &-\lambda_b(\lambda - \frac{1}{2}\lambda_b)\sum_{i=0}^{m}\xi_i\mathcal{C}_{ik} = 0\ ,\quad k = 1\dots m\end{aligned} \tag{1h}$$

which is an algebraic nonlinear system of m equations in the $m+1$ variables ξ_0, $\xi_1\cdots\xi_m$, with known coefficients.

The implementation of the asymptotic approach is quite easy and its computational cost remains of the order of that required by a standard linearized stability analysis [3]. Once the preprocessor phase of the analysis has been performed (steps 1 to 4), the presence imperfections can be taken into account in step 5, by adding additional imperfection terms in the expression of $\mu_k[\lambda]$, allowing for an inexpensive imperfection sensitivity analysis.

3 Imperfection sensitivity analysis

In the analysis of thin walled structures the characterization of imperfection is often difficult. The presence of imperfections changes some aspects of structural response and often causes an erosion of the carrying capacity, especially in the interactive buckling range [9, 10, 11]

In the asymptotic algorithm the presence of imperfections expressed by a load $\tilde{p}[\lambda]$ and/or an initial displacement $\tilde{u}$ affect Eq.(1g) only in the imperfection term $\mu_k[\lambda]$ that becomes (see [3])

$$\mu_k[\lambda] = \frac{1}{2}\lambda_b(\lambda - \frac{1}{2}\lambda_b)\Phi_b'''\hat{u}^2\dot{v}_k + \frac{1}{6}\lambda_b^2(\lambda_b - 3\lambda)\Phi_b''''\hat{u}^3\dot{v}_k + \mu_k^l[\lambda] + \mu_k^g[\lambda] \tag{2}$$

with

$$\mu_k^l[\lambda] + \mu_k^g[\lambda] = \lambda\,(\Phi_c'''\hat{u}\tilde{u}\dot{v}_k - \tilde{p}[\lambda]) = \lambda\bar{\mu}_k \tag{3}$$

The aim of the *imperfection sensitivity analysis* is to link the presence of geometrical and load imperfections to the reduction in the limit load. For structures presenting coupled buckling even a small imperfection in loading or geometry can mean a marked reduction in collapse load with respect to the bifurcation load [12, 13, 14]. So an effective safety analysis should include an investigation of all possible imperfection shapes and sizes to identify the worst cases.

The asymptotic approach provides a powerful tool for performing this extensive investigation. In fact, the analysis for a different imperfection only needs to update the imperfection factors $\mu_k^g[\lambda]$ and $\mu_k^l[\lambda]$ through Eqs.(2)–(3) and solve the nonlinear system (1g)–(1h). Even if this system, which collects all the nonlinear parts of the original problem, proves to be highly nonlinear and some care has to be taken in treating the occurrence of multiple singularities, its solution through a path–following process is relatively easy because of the small number m of unknowns involved.

4 Numerical results

The imperfection sensitivity analysis of the cylinder called Z15 [15, 16] in compression is investigated. The geometrical data and mechanical properties are reported in Table 1. Different thickness have been considered. The boundary condition used is clamped on bottom and top edge except the longitudinal displacement at the top edge. The load is a distributed line load applied at the top edge. The elements used for the mesh have a dimension of $5.8\,x5.8\,mm$. From computational point of view, the Koiter's asymptotic analysis has been performed using corotational approach [17] within a mixed formulation based on MISS-4 finite element [6]. Further details about the implementation can be also found in paper [1].

Z15 Cylinder Geometry		UD CFRP Material Properties	
Length $[mm]$	500.00	$E_1\,[GPa]$	157.4
Radius $[mm]$	250.27	$E_2\,[GPa]$	8.6
Thickness $[mm]$	0.463	$G_{12}\,[GPa]$	5.3
Lay-up [in-out]	$[\pm 24/\pm 41]$	ν_{12}	0.28

Table 1: Geometrical data and mechanical properties.

5 Buckling and post-buckling analyses

The buckling modes are reported in Table 2. Eight buckling modes have been considered for the multimodal analysis. The buckling modes for the the thickness $t = 0.463\ mm$ are reported in Fig. 1. Some of the quadratic corrections (eq.1d) are shown in Figure 2 and used to recover the structural behaviour of the imperfect structure.

Only geometrical imperfections $\tilde{u}$ have been considered. In particular, they are generated as linear combinations of the buckling modes $\dot{v}_i$, that is

$$\tilde{u} = \sum_{i}^{m} r_i \dot{v}_i \tag{4}$$

where r_i are random numbers, and m is the number of buckling modes included in the expansion (1e). For this example, $m = 8$ is used. Note that our aim is to find the worst imperfection. Then, the real shape of the imperfection is not required and only the linear combinations of buckling modes are considered. The maximum value of $\tilde{u}_{max}$ is assumed to be bound by a tolerance

$$\tilde{u}_{max}/t \leq tol \tag{5}$$

For this example, $\tilde{u}_{max}/t = 0.3$ is used, while in practice the amplitude of the imperfection depends on the manufacturing process. The shape of the worst imperfection and the deformed shape at minimum limit load are shown in Figures 3 and 4, respectively. The distribution of limit load λ_{lim} considering one thousand imperfections is shown in Fig. 5

The load sensitivity as a function of the amplitude of the worst imperfection is reported in Figure 6, where it can be seen that cylinder Z15 is highly imperfection sensitive.

thick	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8
$1/4\ t$	1745	1745	1756	1756	1758	1758	1762	1762
$1/2\ t$	7120	7120	7131	7131	7189	7189	7201	7201
$3/4\ t$	16302	16302	16531	16531	16557	16557	16586	16586
t	29556	29556	29945	29945	30059	30059	30199	30199
$5/4\ t$	46918	46918	47740	47740	48117	48117	48194	48194
$3/2\ t$	68848	68848	69324	69324	70614	70614	71499	71499
$7/4\ t$	94844	94844	96862	96862	97115	97115	98470	98470

Table 2: Buckling loads (in Newtons) for cylinder Z15 and various values of the thickness.

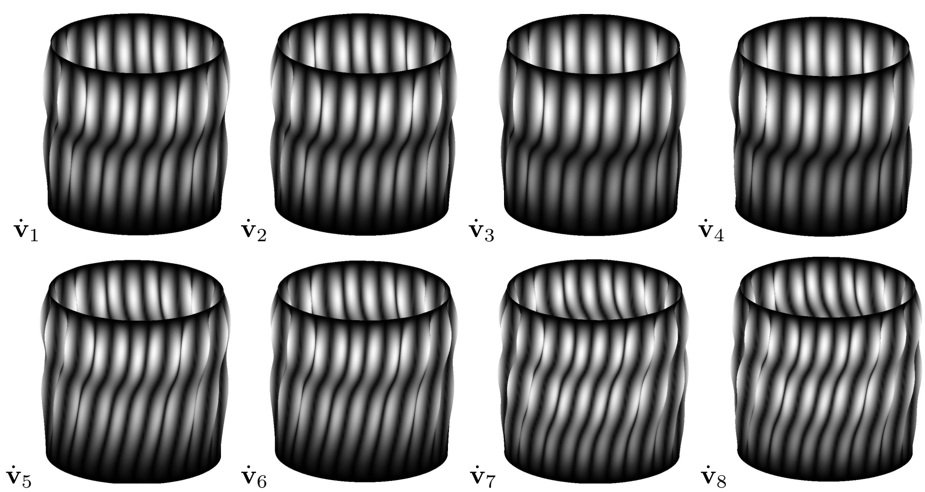

Figure 1: Buckling modes for cylinder Z15 with $t = 0.463\ mm$.

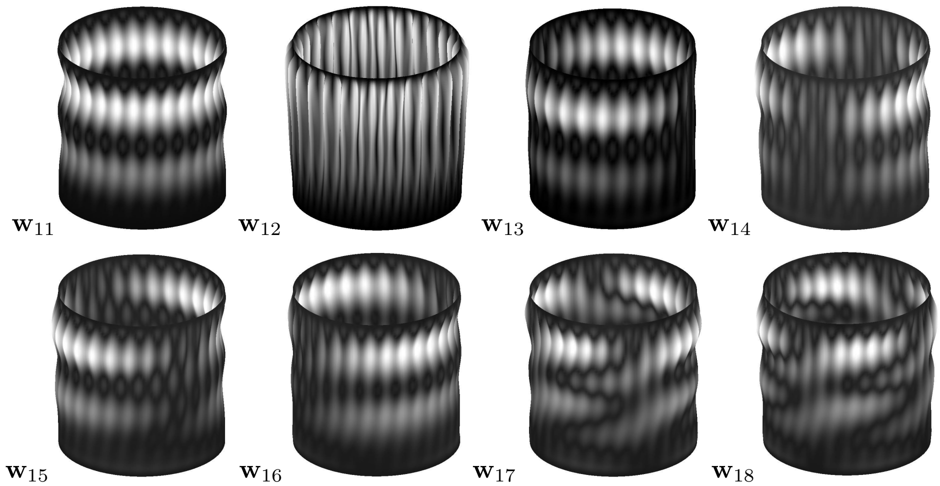

Figure 2: Quadratic corrections for cylinder Z15 with $t = 0.463\ mm$.

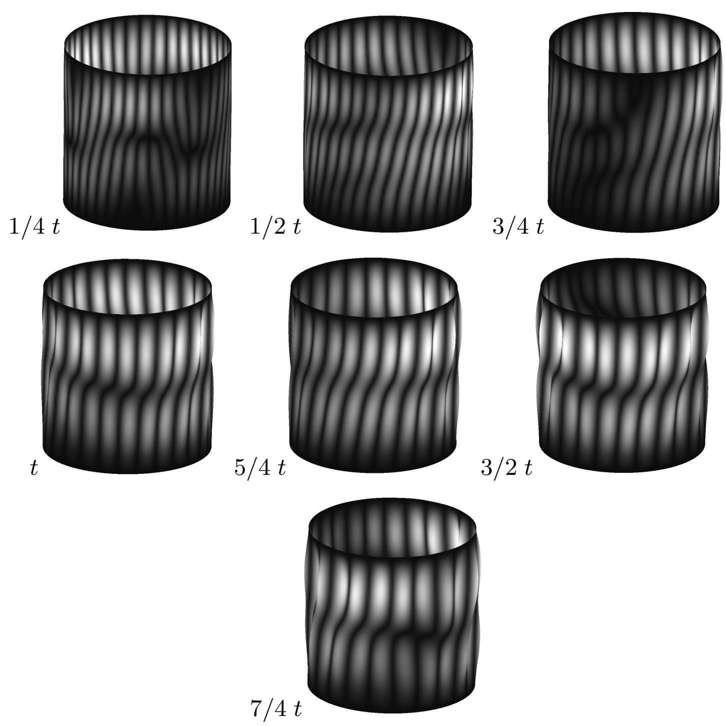

Figure 3: Shapes of the worst imperfection for cylinder Z15 and various values of the thickness.

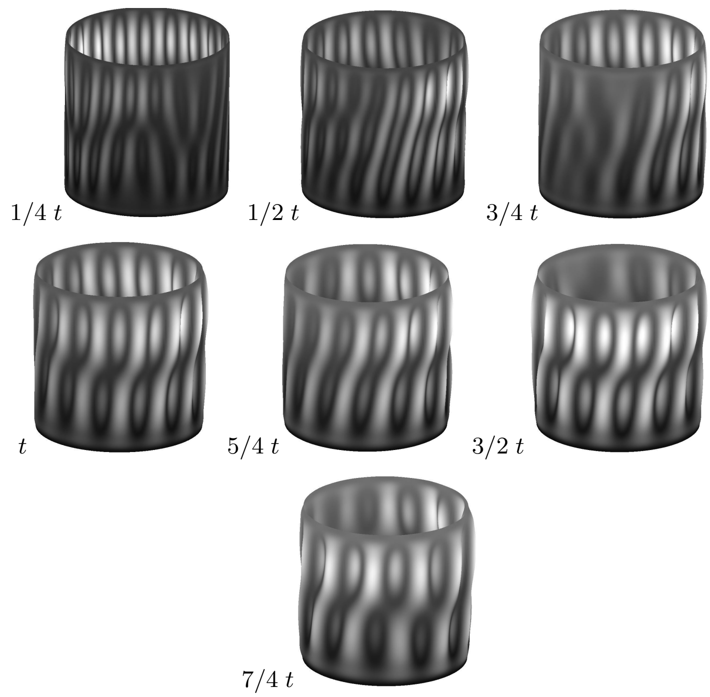

Figure 4: Mode shapes at minimum limit load with the worst imperfection for cylinder Z15 and various values of the thickness.

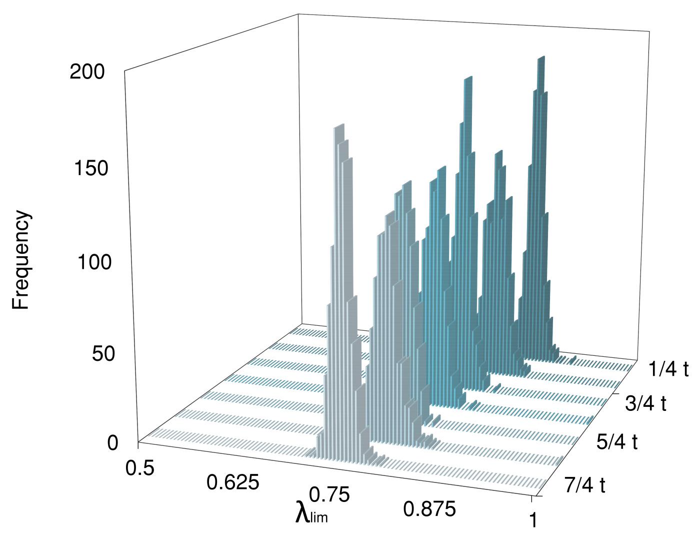

Figure 5: Frequency distribution of the lowest limit load λ_{lim} for cylinder Z15 and various values of the thickness.

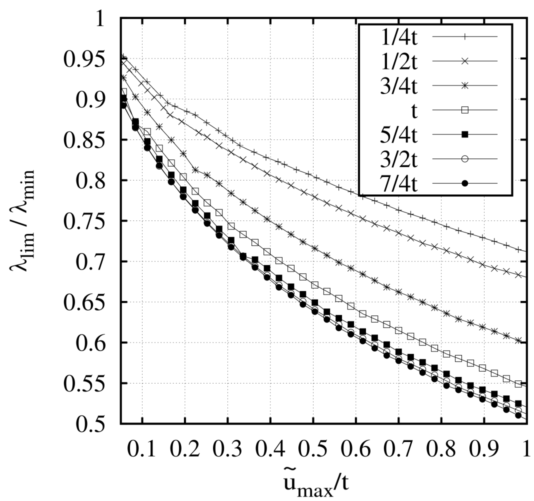

Figure 6: Limit load λ_{lim} normalized on the lower buckling load λ_{min}. Sensitivity to worst imperfection amplitude for cylinder Z15 and several values of the thickness.

6 Concluding remarks

An imperfection sensitivity analysis using Koiter's approach and the Monte Carlo method has been applied for the evaluation of imperfection sensitivity of composite cylindrical shells. The analysis allows to evaluate the limit loads and the erosion of the theoretical buckling due to both imperfections and mode interaction. The main strengths of the proposed methodology are the ability to analyse thousands of random imperfections in a short time, with very low computational cost, to find the worst imperfection and provide an accurate evaluation of limit load and erosion of buckling load, with respect to theoretical case, due to buckling mode interaction. In particular, the overall analysis allows to obtain a statistical evaluation of limit load distribution and the sensitivity curves with respect to worst imperfections. The robustness and the reliability of the methodology confirms that Koiter's approach is a powerful tool for the evaluation of limit performance of composite cylindrical shells.

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