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Reliability Design Methodology for Confined High Pressure Inflatable Structures

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abstract

A design methodology for high pressure, inflatable structures is proposed. The inflatable structure may be partially confined inside large diameter conduits and tunnels. The design addresses the main structural requirements of the system, namely, fabric strength, geometric stability, and axial stability. The proposed design methodology is based on the concept of limit states. Load and resistance factors are identified for all the stochastic variables participating in the structural design equations. A formal methodology is used to estimate the load and resistance factors from known distributions of data for each of the stochastic variables. The concepts of basis values, coverage, and confidence are used along with the analytical treatment necessary to estimate the load and resistance factors. The analysis is applied to the cases of Normal, Log-normal, and Weibull distributions of data. Material characterization and data analysis are presented for fabric strength and friction coefficient between the inflatable and the confining conduit material. The system reliability is also evaluated.

keyword

Protective structures; Confined structures; Inflatable structures; Structural Reliability; Soft good materials, Fabric materials, Textile materials.

1 Introduction

Occasionally, large diameter conduits and tunnels need to be sealed to prevent the flow of liquids or gases in case of emergency, maintenance, or environmental remediation. Applications include large water and sewage pipes, large conduits for industrial fluids, rail and automobile tunnels, and so on, that might be vulnerable to unexpected system failures that require temporary closure. For example, flooding of a freight tunnel in downtown Chicago and the buildings connected to these tunnels, forced evacuation of more than 250,000 people [1]. The Chicago tunnels have a cross section of 2.8 x 1.3 m and at the time of the accident they represented a network of more than 80 km that was used to run freight, television, telephone, and power conduits. Pumping water from the tunnel system took five and a half weeks at a cost of $40 million [2]. Although it is difficult to prevent all situations that can lead to such threatening events, damage can be substantially minimized by compartmentalizing the region affected by the event.

Inflatable structures, such as the one shown in Figure 1, have been proposed [3–5] for sealing large diameter conduits when it becomes necessary to block the flow of a pressurized liquid through the conduit as shown schematically in Figure 2. Typically, the inflatable would be placed inside the conduit compactly folded inside a container (Fig. 1 -a) that holds the inflatable until a signal triggers the deployment (Fig. 1 -b); then, once the plug has deployed, the inflation process begins until the inflatable reaches its full shape (Figs. 1 -c and -d); when the plug has reached the final shape, the pressurization starts and the inflatable is

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able to withstand the external pressure exerted by pressurized liquid acting on one side of the inflatable as illustrated in Figure 2. The geometry is such that the inflatable fits precisely the internal geometry of the conduit to be sealed, which is not necessarily that of a perfect cylinder. In addition to fitting the conduit, the geometry of the in inflatable includes the geometry of the end caps, which are to be designed to minimize the stress on the fabric.

The selection of the fabric hinges on the strength of the fabric and the strength of the welded, bonded, or heat-sealed fabric. Welds are necessary because the geometry of the inflatable is obtained by joining pieces of fabric cut appropriately to yield the desired geometry. Anchorage can be achieved by mechanical anchors or by friction between the inflatable and the conduit. The former introduces undesirably high tear stresses on the fabric at the anchorage points, requires intrusive preparation of the conduit via installation of anchors, and introduces additional modes of failures, including failure of the anchors and the anchoring points in the conduit. When a friction anchoring is used, the friction coefficient between the fabric and the conduit becomes a system property that needs to be characterized. In summary, the lowest fabric strength, welded or un-welded, plus the friction coefficient are the two *resistances* needed for the design.

Regarding the externally imposed loads, the pressure of the fluid to be contained p_e is specified by the particular application at hand. The inflation pressure p_i is a design parameter to be chosen by the designer to seek safe operation of the system. For the design presented here, the pressure p_e is assumed to be applied quasi-statically. No dynamic effects attributed to a sudden rush of fluid are explicitly considered in the design since the normal operation of the inflatable structure is expected to be carried out under quasi-static loading conditions. The importance factor used in the proposed design implicitly accounts for unexpected or unaccounted loading circumstances such as rushing fluid, among others. We acknowledge that the dynamic effects of a rushing fluid may affect the behavior of the structure at the early stages of operation, but that situation ruled out for the range of operating conditions considered in the present work.

While most inflatable structures in use today are inflated at low pressures (e.g., below 0.07 bars) the high pressure inflatables considered in this work are required to operate at substantially higher pressure, which is dependent on the external pressure that they must resist. For this reason, the inflatables discussed in this work are assumed to be constructed of a textile fabric capable of withstanding the stress imposed by the high inflation and external pressures. Such fabric is typically coated with a polymer to provide hermeticity but non-structural bladders may also be used for that purpose.

2 Limit States Design

Common practice for the design of inflatable structures uses Allowable Stress Design (ASD) with large factors of safety to account for uncertainties. On the other hand, general procedures for Limit States Design have been developed for steel, concrete, wood, and masonry structures, for which significant data on performance under various loads, and load combinations exists, which have allowed resistance factors to be clearly defined. This is not the case for inflatable structures. The approach proposed herein is an attempt to improve the current methodology used by softgoods design engineers, whom for the most part are not familiar with reliability design.

The limit states design *(LSD)* method requires only one value to describe each stochastic variable (load, resistances, geometry, and so on). This single value is the *basis* value $x_{p,q}$, which for resistances, defines the interval $(x_{p,q}, \infty)$ that contains a fixed proportion $f < 1$ of the population. In other words, it can be said with confidence q that $100f\,\%$ of the strength data will be above the basis value and that only $100p\,\%$ of the data will be below it, with $p = 1 - f$.

Since tolerance intervals are based on a sample $\mathbf{x}$ containing only n data points out of the entire population, the former assessment can be made only with a certain level of confidence $q < 1$. In the aerospace industry, A-basis is defined with $f = 0.99, q = 0.95$ and B-basis is defined with $f = 0.90, q = 0.95$. For some applications, such as wind turbine blades and marine structures, it is common practice to use a single tolerance interval, namely 95% coverage and 95% confidence [9–11]. In this work, the basis values obtained with 95% coverage and 95% confidence are called C-basis.

Unlike allowable stress design (ASD), LSD uses multiple (partial) safety factors to take into account the uncertainty of each of the parameters separately (loads and resistances) [12,13]. This is in contrast to ASD where a single safety factor is used to safeguard against all the uncertainties without recognizing that various parameters may have drastically different uncertainties.

Although the basis values $x_{p,q}$ for each variable (loads and resistances) are the only values needed for the analysis, it is customary in LSD to use mean values $\bar{x}$ and partial factors ϕ; one pair for each load and resistance. This is equivalent to splitting the basis value into a mean value of resistance R and a resistance factor φ_R using the definition

$$x_{p,q} = \varphi_R \; R \qquad \text{with} \qquad \varphi_R < 1 \tag{1}$$

For the loads, the upper tail of the distribution is used. In other words, the interval $(x_{p,q}, \infty)$ contains a fixed proportion $p = 1 - f$ of the population. Then, it can be said with confidence q that $100f\,\%$ of the load data will be below the basis value and that only $100p\,\%$ of the data will be above it. Again, the basis value is split into the the mean value of load L and a load factor α_L using the definition

$$x_{p,q} = \alpha_L \; L \qquad \text{with} \qquad \alpha_L > 1 \tag{2}$$

Furthermore, LSD recognizes two limit states: serviceability and ultimate. The inflatable structure is designed for the ultimate limit state [12,13]. That is, the structure is designed to safely sustain the loads that might cause failure of the inflatable system. The inflatable structure is not designed for serviceability criteria because the inflatable is not a structure meant to work in service (i.e., permanent condition), but rather it is a safety device that will deploy only during extraordinary events that are triggered by an emergency.

According to LSD, there are two complementary aspects to be considered separately: the resistances of the structure and the loads applied to it. Since there is no available LSD procedure for inflatable structures, as a starting point for this work, it is proposed that design must satisfy the following inequality [12]

$$\varphi_R R > \alpha_D D + \psi\gamma \left\{ \alpha_L L + \alpha_T T \right\} \tag{3}$$

where

$$\begin{aligned}
\varphi_R &= \text{Resistance Factor}\\
R &= \text{Material Resistance}\\
D &= \text{Dead Load}\\
L &= \text{Live Load}\\
T &= \text{Thermal Load}\\
\alpha_D &= \text{Dead Load Factor}\\
\alpha_L &= \text{Live Load Factor}\\
\alpha_T &= \text{Thermal Effect (Temperature) Load Factor}\\
\psi &= \text{Load Combination Factor}\\
\gamma &= \text{Importance Factor}
\end{aligned}$$

As shown in Figure 2, there are two main loads acting over the inflatable structure, the inflation pressure and the external pressure. Both of these are live[3] loads. There are no dead loads[4]. No thermal loads have been defined for this application, but they may be included in the future if the fluids being transported by the conduit are not at room temperature.

The two loads applied to the structure define three different limit states that must be verified using (3). These ensure that the structural strength of the material is sufficient to resist the stresses, that the axial stability of the structure is maintained, and an additional functional requirement that relates the internal and external pressure applied to the inflatable plug.

For this particular application, the importance factor is determined by the criticality of failure of the system and the existence or not of redundant loads paths to alleviate unexpected circumstances. In addition, for this particular application, *customer's specifications* require us to include an factor γ to account for the consequences of the system failing due to unexpected or unaccounted circumstances. The influence of the importance factor value chosen on the system reliability in evaluated in § 5.

[3]Unlike dead loads, live loads do not act permanently on the structure but rather on occasion, due to the normal operation or function of the structure.

[4]Dead loads, such as self weight, act permanently on the structure.

2.1 Determination of the Inflation Pressure

The inflation pressure p_i must be larger than the fluid pressure p_e to maintain the upstream end cap geometrically stable (Figure 2). Furthermore, if p_i drops closer to p_e, the fluid can seep around the inflatable creating a layer that reduces the contact between the inflatable and the conduit walls drastically, thus compromising the anchoring force, which is entirely due to friction. On the other hand, the inflation pressure should be as small as possible to minimize the stresses in the fabric on the downstream cap (Figure 2). This latter requirement is now formalized as follows:

$$\varphi_{pi}\ p_i > \psi\gamma\alpha_L p_e \tag{4}$$

Note that the inflation pressure is on the *resistance* side of the inequality (4) and that a reduction of that pressure would be detrimental. Therefore, the inflation pressure is multiplied by a *reduction* factor $\varphi_{pi} \le 1$. This is in contrast to the fluid pressure p_e, which is on the *load* side of the inequality (4) and thus it is affected by a load factor $\alpha_L \ge 1$.

The *resistance factor* φ_{pi} is determined by the variability in the inflation pressure p_i, which is maintained by a pump, valve, and control system with variability described by a Normal distribution. Therefore, the basis value for the resistance factor φ_{pi} is calculated according to the methodology presented in [14] as:

$$x_{p,q} = \bar{x} - k_{p,q}(n)\ s \tag{5}$$

where $\bar{x}, s$, are the mean value and standard deviation of the sample data, and $k_{p,q}$ is tabulated in [14, Table 1] as a function of the selected coverage f, confidence q, and sample size n. Usually, a sufficiently large data set ($n \to \infty$) exists for the pressure provided and controlled by inflation system, which allows one to assume that the sample mean $\bar{x}$ and standard deviation s approach the population mean $\bar{\bar{x}}$ and variance σ. In other words, the population mean and variance are known with high confidence (i.e., $q = 1$), so that $\bar{x} \to \bar{\bar{x}}$ and $s \to \sigma$. In this case, discarding the lower tail of the distribution:

$$k_{p,1} = \Phi^{-1}(f) \tag{6}$$

that is, the factor can be calculated simply as the value of the inverse cumulative distribution function in standard form [15, 16], which is widely available in tabular form, or using the MATLAB® function *icdf('normal',f,0,1)*, where f is the coverage (e.g., $f = 0.95$ for C-basis). Combining (2,5,6), the resistance factor can be calculated as:

$$\varphi_{p_i} = 1 - \Phi^{-1}(f)\, C_V \tag{7}$$

where $C_V = s/\bar{x}$ is the coefficient of variation (COV).

The *load factor* α_L to be applied on p_e is chosen taking into account that the upstream pressure is a live load [17]. The upstream pressure is due to the fluid head of a reservoir with variability typically described by a Normal distribution. Then, the basis value is calculated according to (5). Usually, a sufficiently large data set ($n \to \infty$) exists for the reservoir head (i.e., pressure head due to the level of the reservoir as a function of time, from historical records). That allows one to assume that the population mean and variance are known with high confidence (i.e., $q = 1$). In this case, discarding the higher tail of the distribution

$$k_{p,1} = \Phi^{-1}(p) \tag{8}$$

that is, the factor can be calculated simply as the value of the inverse cumulative distribution function in standard form [15, 16], which is widely available in tabular form, or using the MATLAB® function *icdf('normal',p,0,1)*, where $p = 1 - f$, and f is the desired coverage (e.g., $f = 0.95, p = 0.05$, for C-basis). Combining (2,5,8) and noting that $\Phi^{-1}(p) = -\Phi^{-1}(f)$, the load factor can be calculated for Normally distributed data as

$$\alpha_L = 1 + \Phi^{-1}(f)\, C_V \tag{9}$$

Since there is no dead load, the load combination factor is $\psi = 1.0$.

The importance factor γ is chosen taking into account that violating (4) may result in partial loss of frictional anchoring force, which may partially imperil the functionality of the system. The selection of the importance factor for the inflation pressure is based on the ability of the system to recover from unexpected loss of pressurization, which is typically achieved having redundant detection and inflation systems that can back up the primary systems in case of malfunction and that are activated before complete depressurization of the inflatable. The importance factor γ also takes into account whether on not redundant features and/or redundant load paths exist that can mitigate the effect of unexpected overloads. Fabric failure may be caused by an unexpected rise of fabric stresses, for example due to unaccounted stress concentrations attributed to plug misalignment, plug distortions, lack of conformance to the conduit and so on, which if they exceed the strength of the material would lead to a catastrophic failure of the whole system. Also, unexpected changes in the surface roughness and/or cleanliness of the conduit's surface leading to changes in the friction coefficient, would compromise the axial stability, meaning that the inflatable would slide along the conduit, with the consequent catastrophic failure of the system.

Finally, the required inflation pressure is calculated from (4) as

$$p_i > \frac{\gamma \alpha_L p_e}{\varphi_{p_i}} \tag{10}$$

2.2 Design for Material Strength

An inflation pressure p_i is applied to inflate the structure sufficiently for it to maintain its shape and to generate enough friction against the conduit surface to be able to prevent the inflatable from sliding along the conduit under the action of the upstream fluid pressure p_e.

On the portion of the inflatable that is in contact with the wall (Figure 3), heretofore called the cylindrical portion, the wall equilibrates the inflation pressure exactly and the fabric is subject to negligible stress. On the cylindrical portion, the fabric acts only as an impervious media and helps contain the inflation fluid during the inflation process, but once inflated, the fabric in the cylindrical portion is subjected to no appreciable stress. On the upstream side, the fabric is subjected to the stress necessary to equilibrate the pressure differential between the upstream pressure p_e and the inflation pressure p_i. On the downstream side, the fabric holds the stress needed to equilibrate the entirety of the inflation pressure p_i. For this reason, p_i is chosen as small as possible.

For calculation of stress in the fabric, the fabric can be assumed to be inextensible. This is a good approximation for the type of fabrics used and the pressures that can be applied to them. Therefore, the geometry does not change appreciably as a result of inflation and the calculation of stress in the fabric is that of an isostatic structure. The geometry of the membrane has, at every point, two principal radii of curvature r_1 and r_2. According to Laplace's equation [18], the largest principal stress-resultant is

$$N = t\sigma = \frac{\Delta P}{(1/r_1 + 1/r_2)} \tag{11}$$

where t is the thickness of the fabric and ΔP is the pressure differential across the membrane (i.e., the pressure differential contained by the membrane). For fabrics, it is customary to use stress-resultants $N = t\sigma$ in N/m instead of stress σ. For a sphere, $r_1 = r_2 = r$, and for a cylinder, $r_1 = r, r_2 = \infty$, so

$$N^{sphere} = \frac{r}{2}\Delta P \tag{12}$$

$$N^{cyl} = r\ \Delta P \tag{13}$$

Comparing (12) with (13), one concludes that the stress-resultant in a membrane of general curvature is bracketed by the values for the sphere and cylinder, with the value for the cylinder being the maximum possible. In our current application, Figure 3 shows that the loads in the cylinder are directly transferred to the conduit surface and, therefore, the maximum fabric stress-resultant is found at the downstream sphere, which is defined by equation (12), with $\Delta P = p_i$, where p_i is defined by (10).

According to the limit state equation (3),

$$\varphi_R R > \psi \alpha_L N \tag{14}$$

where R is the mean value of resistance of the fabric and φ_R is the resistance factor associated to the value of R. This factor depends on the variability of R. The fabric strength data is analyzed in § 3.

The stress-resultant N is calculated using (12). The load factor α_L to be applied on p_i in (14) is chosen taking into account that the inflation pressure is a live load with variability given by the inflation system (pumps, regulators).

Since there is no dead load, the load combination factor is chosen to be $\psi = 1.0$. From (12 - 14) one can calculate the required fabric resistance as

$$R > \frac{\alpha_L \, r \, p_i}{2 \, \varphi_R} \tag{15}$$

2.3 Design for Axial Stability

The conduit is anticipated to a have a prismatic cross section, not necessarily circular, with perimeter S. The inflatable geometry mimics the prismatic geometry of the conduit and it is in contact with it over a contact length L_c, defined by the length of the prismatic section. Therefore, the axial force that friction can sustain can be calculated as follows:

$$F_r = \mu p_i S L_c \tag{16}$$

where μ, p_i, S, L_c, are the friction coefficient, inflation pressure, perimeter, and contact length, respectively. For a circular conduit of radius r, the perimeter is $S = 2\pi r$.

The applied axial force is caused by the pressure differential between the upstream pressure p_e and downstream pressure p_o. Assuming atmospheric downstream pressure, the applied axial load is

$$F_a = p_e \; A \tag{17}$$

where A is the cross-sectional area of the conduit. Both A and S are assumed to be deterministic parameters (i.e., they have no variability). For a circular conduit of radius r, the area is $A = \pi r^2$.

According to the LSD equation (3),

$$\varphi_F F_r > \psi \; \alpha_L F_a \tag{18}$$

From (16 - 18), the following limit state requirement is obtained

$$\varphi_\mu \; \mu \; \varphi_{pi} \; p_i \; S \; L \; > \psi \; \alpha_{pe} \; p_e \; A \tag{19}$$

where p_i is defined by (10) and the resistance factor φ_μ is determined by the variability of the friction coefficient μ, analyzed in § 3.

Since there is no dead load, the combination factor is chosen again as $\psi = 1$.

From (19) one can calculate the contact length required for the inflatable as

$$L_c > \left(\frac{\alpha_{pe} \; p_e}{\varphi_{pi} \; p_i} \right) \left(\frac{r}{2 \; \varphi_\mu \; \mu} \right) \tag{20}$$

3 Material Characterization

According to LSD [17], there are two complementary aspects to be considered: the resistances of the structure and the loads applied to it. There are two material/system properties, or *resistances*, that control the resistance aspect of the design: the tensile strength of the fabric R and the friction coefficient μ between fabric and conduit.

3.1 Tensile Strength of the Fabric

A variety of fabrics can be used for inflatable structures as long as they are coated by a polymer. The coating serves two purposes. First, it makes the fabric impervious to contain the inflation pressure. Second, it provides a weldable substance, so that cut pieces of fabric can be welded together into the desired shape. Other joining techniques, such as stitching, do not provide as high load transfer as welding, and they are slower and more expensive to execute. Un-coated fabrics can still be welded by local insertion of a weldable polymer that impregnates and welds the fabric but the un-coated fabric would lack imperviousness. Fabrics can be coated on one or both sides. When coated on one side only, a strip of coating needs to be used to provide enough weldable polymer to impregnate and weld effectively the un-coated side of the fabric.

Four types of fabrics were considered for manufacturing the prototype designed in this work. The main properties of the each fabric considered are summarized in Table 1. The most important requirements for the design are that the prototype is required to sustain the membrane stresses exerted by the inflation and fluid pressure, as well as to provide acceptable friction properties to assure axial stability of the structure. After preliminary evaluation of the options shown in Table 1, the fabric chosen for the manufacturing of the prototype designed in this work is a woven fabric, constructed of Vectran HS fibers, 1500 denier with 4x4 basket weave pattern. This fabric is characterized by a total of 34 yarns per 25.4 mm the warp direction and 42 yarns per 25.4 mm in the fill direction. The thickness is 0.838 mm. This fabric had also a urethane polymer coating on both sides [19, 20].

Tensile strength of fabrics R is reported as a force per unit width (i.e., $[N/mm]$). Tensile tests were performed on samples of fabrics to generate enough data to support the development of the resistance factor used in the design. Previous research [21] have demonstrated that the biaxial strength of coated fabrics is equal to the uniaxial strength. Therefore, the tensile strength of the material used in this work was characterized by standard uniaxial tests. Specifically, tensile tests were performed according to ASTM D 5035 [22], on a universal testing machine, model MTS 318.10 with hydraulic wedge grips specialized to hold fabrics, model MTS 647.10A [20]. The data is analyzed in Table 2 for the purpose of this study.

Fabric strength is highly dependent on fiber strength, which is known to be best represented by a Weibull distribution. Therefore, the basis value is calculated using [14]

$$x_{p,q} = \hat{\lambda}\left[-\ln(1-p)\right]^{1/\hat{k}} \exp\left(\frac{-V_{p,q,n}}{\hat{k}\sqrt{n}}\right) \tag{21}$$

where $\hat{\lambda}, \hat{k}$ are the maximum likelihood estimates (mle) for the data and $V_{p,q,n}$ is tabulated in [14] as a function of the coverage f, confidence q, and sample size n. The calculations are summarized in Table 2, with the *mle* values calculated using the MATLAB function *mle(x,'distribution','weibull')*, and x is the data.

From Table 2, the lowest basis value is $x_{95,95} = 318.532\,N/mm$ for the warp direction of the fabric, corresponding to a mean value $\bar{R} = 406.207\,N/mm$ and resistance factor $\varphi_R = 0.784$.

3.2 Friction Coefficient

The friction coefficient μ is a system property that controls the amount of axial force that can be obtained from a given inflation pressure. It is assumed that the inflatable is in contact with the internal surface of the conduit, which could be concrete, masonry, steel, or other. The surface could be smooth or rough depending on the location where the inflatable is installed. Also, the fabric and the surface of the conduit may be dry or wet when the inflatable is deployed. Tests were performed to investigate these configurations.

The friction coefficient is a property that is strongly dependent on the system configuration [23]; that is, type of surfaces, combination of loads applied to the surfaces, lubrication effect of liquids if present, and so on. Therefore, a custom test set up was developed for determination of the friction coefficient [19]. The experimental apparatus is shown in Figure 4. A hydraulic pump (1) pushes a car with a concrete block on it (2). The car slides below a piece of fabric (3) that is held by the vertical loading ram, loaded with a known weight (4). The square fabric specimen has a surface area of 12.65 cm^2 and it is held to the flat end of the loading ram by double-side adhesive tape. The horizontal force required to move the car at constant speed is measured by a load cell (5). The displacement of the car is measured with a linear voltage differential transducer (LVDT) (6). The load cell and LVDT output are converted to SI units and recorded on a computer via a data acquisition system.

To reproduce with the maximum possible accuracy the configuration of our system, the vertical load applied to measure the friction coefficient is 309.0 N, which gives a normal pressure of 239 kPa. Friction coefficient data for friction between Urethane-coated 4×4 Vectran and rough and smooth concrete under dry and wet conditions from [20] is analyzed in Table 3 for the purpose of this work.

Since friction cannot be negative, the friction data is assumed to have a log-normal distribution. Therefore, the basis value is calculated using [14], as:

$$x_{p,q} = \exp\left[\bar{x}_{LN} - k_{p,q}(n) s_{LN}\right] \tag{22}$$

where $\bar{x}_{LN}$ is the sample mean in log space[5], s_{LN} is the standard deviation in log space, and $k_{p,q}(n)$ is tabulated in [14] as a function of the coverage f, confidence q, and sample size n. The calculations are summarized in Table 3.

From Table 3, the lowest basis value is $x_{95,95} = 0.375$ for smooth, wet concrete, corresponding to a mean value $\bar{\mu} = 0.617$ and resistance factor $\varphi_\mu = 0.608$. *Note that the lowest basis value governs the design, and not necessarily the lowest mean value.*

4 Prototype Design

In this section, the methodology proposed in § 2 and 3 is applied to the design of a prototype. The specifications for the prototype are:

- Circular cylindrical shape
- Test pipe radius $r_p = 0.61\ m$
- Upstream pressure $p_e = 207\ kPa$

Although the test pipe is circular with radius $r = 0.61\ m$, the radius of the inflatable was chosen slightly larger ($r_i = 0.635\ m$) to ensure that the inflatable fits snugly against the pipe even if some wrinkles develop during inflation.

4.1 Inflation Pressure

The inflation pressure is determined using (10) for a given upstream pressure p_e. The upstream pressure is assumed to have a Normal distribution with COV=5%. Using (8) yields a load factor $\alpha_L = 1.082$. The inflation pressure is assumed to have a Normal distribution with COV=10%. Using (6) yields a resistance factor $\varphi_{p_i} = 0.872$. The importance factor is chosen to be $\gamma = 1.2$ to be sure that the nominal inflation pressure is at least 20% higher than the nominal fluid pressure. Then,

$$p_i > \frac{\gamma \alpha_L p_e}{\varphi_{p_i}} = \frac{1.2 \times 1.082 \times 207}{0.872} = 308.221\, kPa \tag{23}$$

Therefore, the inflation pressure is chosen to be $p_i = 310\ kPa$.

4.2 Fabric Strength

The required material strength R is determined using (15) for a given pipe radius r_p and selected inflation pressure p_i. Since p_i acts as a load, its load factor is calculated with (8), i.e., $\alpha_L = 1.128$.

The strength data collected in this study was analyzed in § 3.1. Considering a C-basis, the resistance factor was calculated to be $\varphi_R = 0.784$.

Since only one load is considered, the load combination factor is $\psi = 1$. Then, from (15), the required fabric strength is

$$R > \frac{\alpha_L\, r\, p_i}{2\varphi_R} = \frac{1.128 \times 0.635 \times 310}{2 \times 0.784} = 141.611\, N/mm \tag{24}$$

From Table 2, the available fabric strength is $R = 406.207\, N/mm$, which is sufficient for this application.

[5] Log space is defined by taking the natural logarithm of the data, then the mean and standard deviation.

4.3 Length of the Inflatable

The length required to develop sufficient axial resistance to prevent the inflatable from sliding under the action of the upstream pressure is calculated with (20). Since the test pipe is cylindrical, the cross section is $A = \pi r^2 = 1.169\ m^2$ and the perimeter is $S = 2\pi\ r = 3.833\ m$.

The friction coefficient data collected in this study was analyzed in § 3.2. The mean value of friction is $\mu = 0.617$. Considering C-basis, the friction resistance factors is $\varphi_\mu = 0.608$.

Since the inflation pressure acts as a resistance, its resistance factor is calculated with (6), i.e., $\varphi_{p_i} = 0.872$. On the other hand, the upstream pressure acts as a load, so its load factor is calculated with (8), i.e., $\alpha_{p_e} = 1.082$. Therefore,

$$L_c > \left(\frac{\alpha_{pe}\ p_e}{\varphi_{pi}\ p_i}\right)\left(\frac{r}{2\ \varphi_\mu\ \mu}\right) = \frac{1.082 \times 207 \times 1.169}{0.608 \times 0.617 \times 0.872 \times 310 \times 3.833} = 0.673\, m \tag{25}$$

Due to manufacturing reasons, the length of the cylindrical portion is chosen to be $L_c = 0.70\ m$.

5 System Reliability

The confined inflatable plug can be considered as a series system. In order to work, the three criteria (i.e. inflation pressure, axial stability, and material strength) have to be met simultaneously. The failure functions that describe this system can be written as

$$g_1 = p_i - p_e \tag{26}$$

$$g_2 = 2\ L_c\ p_i\ \mu - p_e\ r_1 \tag{27}$$

$$g_3 = R - \frac{p_i\ r_2}{2} \tag{28}$$

where g_1, g_2, g_3 are the failure functions for the inflation pressure, axial stability and fabric strength, respectively. The description of each one of the variables involved in the failure functions is given in Table 4. It is assumed that the statistical parameters calculated from experimental data are the parameters of the probability distribution functions for friction and fabric strength.

5.1 Inflation Pressure

Interference theory [26, 27] is used to calculate the probability of failure for the failure function g_1. The function includes a linear combination of two random variables with Normal distribution for which the probability of failure is given by

$$\begin{aligned} \bar{\mu}_{g_1} &= \bar{\mu}_{p_i} - \bar{\mu}_{p_e} \\ \bar{\mu}_{g_1} &= 308.22 - 207 = 101.22 \text{ KPa} \end{aligned} \tag{29}$$

$$\begin{aligned} \sigma_{g_1} &= \sqrt{\sigma_{p_i}^2 + \sigma_{p_e}^2} \\ \sigma_{g_1} &= \sqrt{30.822^2 + 10.35^2} = 32.51 \text{ KPa} \end{aligned} \tag{30}$$

where $\bar{\mu}_{g_1}, \sigma_{g_1}$ are the mean value and standard deviation of the failure function, respectively. Therefore, the probability of failure (i.e. $P\{g_1 \leq 0\}$) for the inflation pressure requirement can be calculated as

$$\begin{aligned} P_{F_{g_1}} &= \Phi(0, \bar{\mu}_{g_3}, \sigma_{g_3}) \\ P_{F_{g_1}} &= \Phi(0, 101.22, 32.51) = 9.354\ 10^{-4} \end{aligned} \tag{31}$$

Then, the reliability for this condition of the system is $\Re_{g_1} = 0.999065$.

5.2 Axial Stability

The axial stability failure function is a non-linear combination of Normal and Log-Normal random variables. For this analysis, three variables are considered as stochastic: the inflation pressure p_i, the friction coefficient μ, and the upstream pressure p_e. The distribution types and parameters used for the computations are listed in Table 4. The probability of failure is computed using the gradient based algorithm proposed by Rackwitz and Flessler [24]. The estimated probability of failure is

$$P_{F_{g_2}} = 2.181\ 10^{-5} \tag{32}$$

which corresponds to a reliability of $\Re_{g_2} = 0.999978$.

5.3 Material Strength

The failure equation that represents material strength is a function of two random variables: the fabric strength R (Weibull distribution) and the inflation pressure p_i (Normal distribution). Applying interference theory [26, 27], the probability of failure is

$$P_{F_{g_3}} = 2.492\ 10^{-13} \tag{33}$$

which corresponds to a reliability of practically $\Re_{g_3} \approx 1.0$.

5.4 Series System

For *series* systems made of n components, the probability of failure of the system P_F, can be bounded using the individual probabilities of failure of each component P_i [28]

$$\max\{P_i\} \leq P_F \leq \sum_{i=1}^{n} P_i \tag{34}$$

where the lower limit is determined by the maximum of the individual probabilities of failure, and the upper limit is defined by the sum of the individual probabilities of failure. Then, substituting the individual probabilities of failure obtained in (31), (32), (33) in (34), yields

$$9.354\ 10^{-4} \leq P_F \leq 9.572\ 10^{-4} \tag{35}$$

Therefore the reliability $\Re$ of the system is bounded by

$$0.999043 \leq \Re \leq 0.999065 \tag{36}$$

5.5 Influence of Importance Factor on System Reliability

It can be seen in (26)–(28), that there are two sets of variables that affect the design. The first set includes the external pressure p_e, the fabric strength R, the friction coefficient μ, and the radius of the confining surface r. Typically, the characteristics of this first set of variables are predetermined and cannot be easily changed. The second set of variables includes the inflation pressure p_i, and the contact length L_c. From this second set, the length L_c is function of the inflation pressure as seen in (25). Therefore, the main variable that the designer can adjust during the design process is the inflation pressure p_i.

The inflation pressure p_i is function of a given upstream pressure p_e and of a selected importance factor γ, as seen in (23). Typical importance factors for structures subjected to wind pressures vary from 1.0 to 1.15 depending of the category of the building [?] or from 1 to 1.5 for seismic design depending of the occupancy category defined by the code [?]. Since there are no codes for the design of confined inflatable structures, it is important to evaluate the influence of the importance factor γ on the individual probabilities of failure described in § 5.2–5.3. Three values of importance factors were chosen to evaluate such influence: $\gamma = 1.0, 1.2, 1.5$, and the reliability (as well as probability of failure) were evaluated for each failure function and for the complete system.

For failure function g_1, the probabilities of failure show that as the importance factor increases, the probability of failure decreases two orders of magnitude for each increase in the importance factor, as summarized in Table 5. Note that for $\gamma = 1.0$ the reliability is 96.41%, which given the criticality of the structure, may be deemed insufficient. However, when $\gamma = 1.2$ the reliability approaches to 99.91%, and for $\gamma = 1.5$ the reliability increases to 99.9996%. Since the later may be unfeasible to achieve from a technical point of view, an importance factor for the pressure of $\gamma = 1.2$ is considered appropriate for this design.

For failure function g_2, the change of the length L_c calculated using (25), as function of different values of p_i, does not seem to affect the reliability as the importance factor increases. For all the values of γ, the reliability values remained in the same order of magnitude, as summarized in Table 6.

For failure function g_3, the stress is calculated using (24) as function of different values of p_i. As expected, the reliability decreases as the importance factor increases because the stress approaches the strength of the fabric, thus reducing the margin of safety. For each increase of the importance factor, the probability of failure increases two orders of magnitude. Despite this increase, the probabilities of failure are still very small and the resultant reliabilities are practically 100%. These results are illustrated in Table 7.

The probability of failure of the system defined in (34) is used again to determine the limits as function of the importance factor. Table 8 summarizes these limits based on the results obtained from the individual failure equations presented in Tables 5–7. As seen in Table 8, the reliability of the system is bounded by a very narrow range for each individual importance factor and increases as the importance factor increases.

A more general approach can be used for the reliability design of confined inflatable structures as proposed by Galambos et al. [6] and Ellingwood et al. [7] based on the classical reliability theory presented by Ang and Allin [8]. This general procedure has been used to determine load combination factors for different reliability indexes that are currently used in design codes such as ACI or AISC. However, implementation of the general methodology for the design of inflatable structures is not a simple task. In the general approach, the random variables with non-Normal distributions require transformation from non-Normal variables to equivalent Normal variables prior to the solution of the limit state equations (26)–(28). Then, the reliability of design can be evaluated by [6–8]

$$\begin{aligned} g\left(\phi_1\mu_1, \cdots, \phi_n\mu_n\right) &= 0 \\ \phi_i &= 1 - \alpha_i\beta\frac{\sigma_i}{\bar{\mu}_i} \\ \alpha_i &= \frac{\sigma_i \frac{\partial g}{\partial x_i}}{\sqrt{\sum\left(\sigma_i\frac{\partial g}{\partial x_i}\right)^2}} \end{aligned} \tag{37}$$

where $\bar{\mu}_i, \sigma_i$, are the mean and standard deviation of μ_i, respectively; β is the target reliability, also called reliability index or safety index. This index, according to Galambos et al. [6] "...may be established by reviewing the levels of reliability inherent in those existing standards which have resulted in the past in satisfactory performance." While target reliabilities are available reinforced concrete, steel, wood, composites, and masonry, there are not available for *confined* inflatable structures and there are very few structures of this type to look at. Thus, there are no target reliabilities available for this new type of structures. The designer would have to assume target values that will need to be confirmed either by experiment or by numerical simulations in order to find optimum or minimum values of β for satisfactory designs.

Note also that the geometrical cap stability equation (26) and the material strength equation (28) are linear equations but the axial stability equation (27) is nonlinear. For the implementation of the general method, the later will require linearization of the failure surface. Otherwise, the selected reliability index β could be affected by the formulation of the limit state equation [6]. Since the factor α_i is a function of the gradient $\partial g/\partial x_i$ and that (27) includes the product of two random variables (μ and p_i), the gradient with respect to one variable depends on the other variable. Therefore, the solution of (37) is not trivial. In summary, the general methodology is more complex and requires information that is not available at this early stage of development of confined inflatable structures, thus making its implementation to exceed the scope of our effort.

6 Conclusions

A reliability based methodology is proposed for the design of *confined* inflatable structures. Determination of resistance factors has been shown to follow naturally from the variability of the experimental data normally gathered in support of this type of design. The limit states design procedure helps the designer pay due consideration to each and every one of the various important aspects that may compromise the safety of the system. While the selection of resistance factors falls mostly on the designer, the selection of load factors and importance factor also involves the customer, helping them focus on what is important. This is not always an easy task, but it is crucial for the success of the application. By analyzing the friction coefficient data, it is shown empirically that the basis value governs the design, rather than the mean or the resistance factor. This further demonstrates that the limit states design methodology in this work is a Level I reliability method, which uses only one value (the basis value) for each stochastic variable in the design equations. Further, it is shown that the basis values can be calculated easily and directly from the experimental data that normally accompanies the design effort for novel applications of inflatables at high pressure, confined situations. The reliability of the design is evaluated in terms of a single importance factor. Results show that although individual probabilities of failure are different for each failure condition and have different orders of magnitude, the system reliability increases as the importance factor increases.

Acknowledgements

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Figures

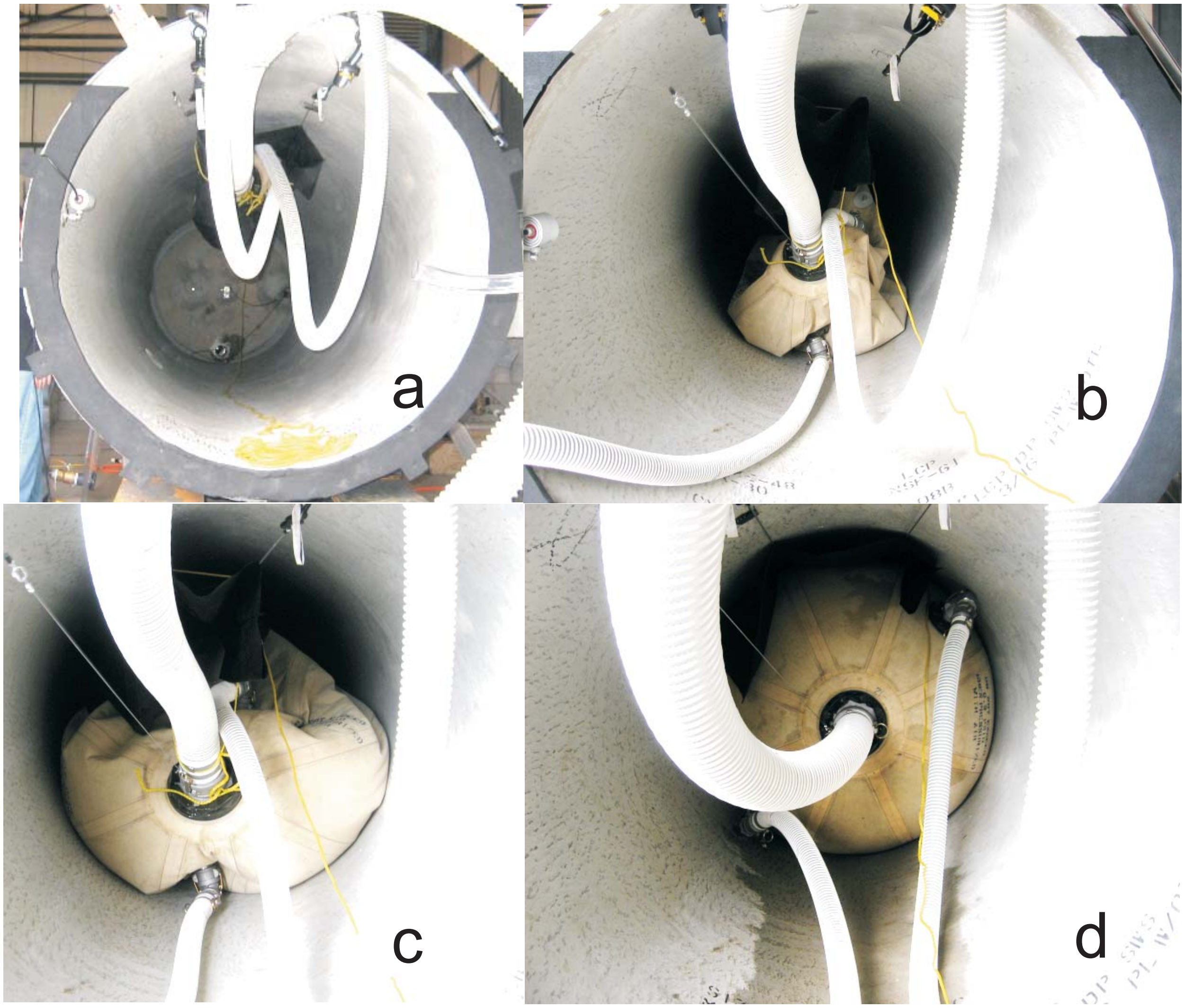

Figure 1: Inflation sequence (left to right). The fabric plug, initially hanging from the roof of the conduit (a), is filled with water through the large diameter hose (b). Any trapped air is released through the small diameter hose on top of the plug (c), which is then closed to allow for full pressurization (d). The small hose on the bottom of the plug is later used for draining the plug.

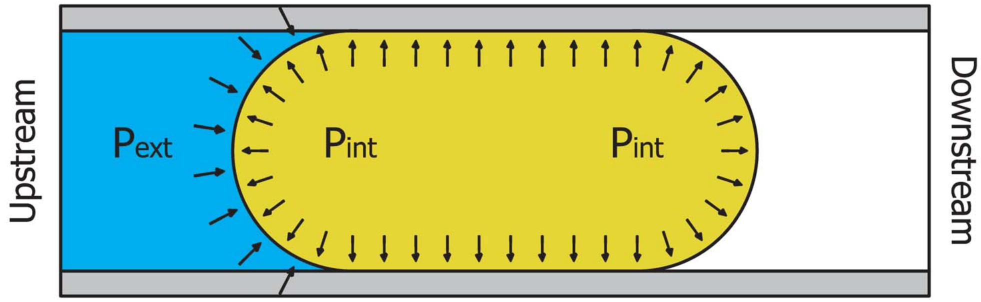

Figure 2: Schematic of the proposed inflatable structure to seal a conduit.

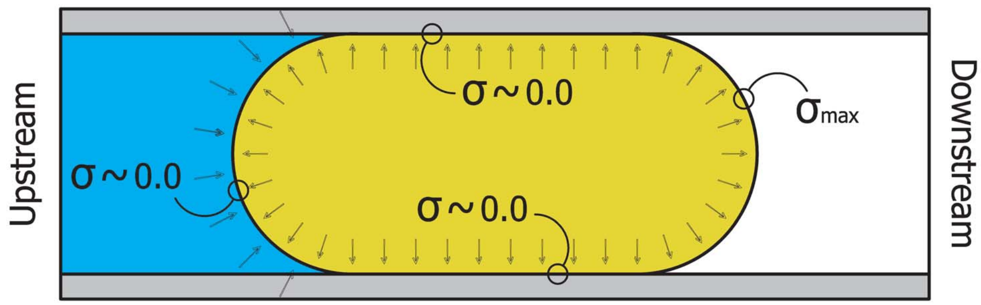

Figure 3: Stresses in the different regions of the inflatable.

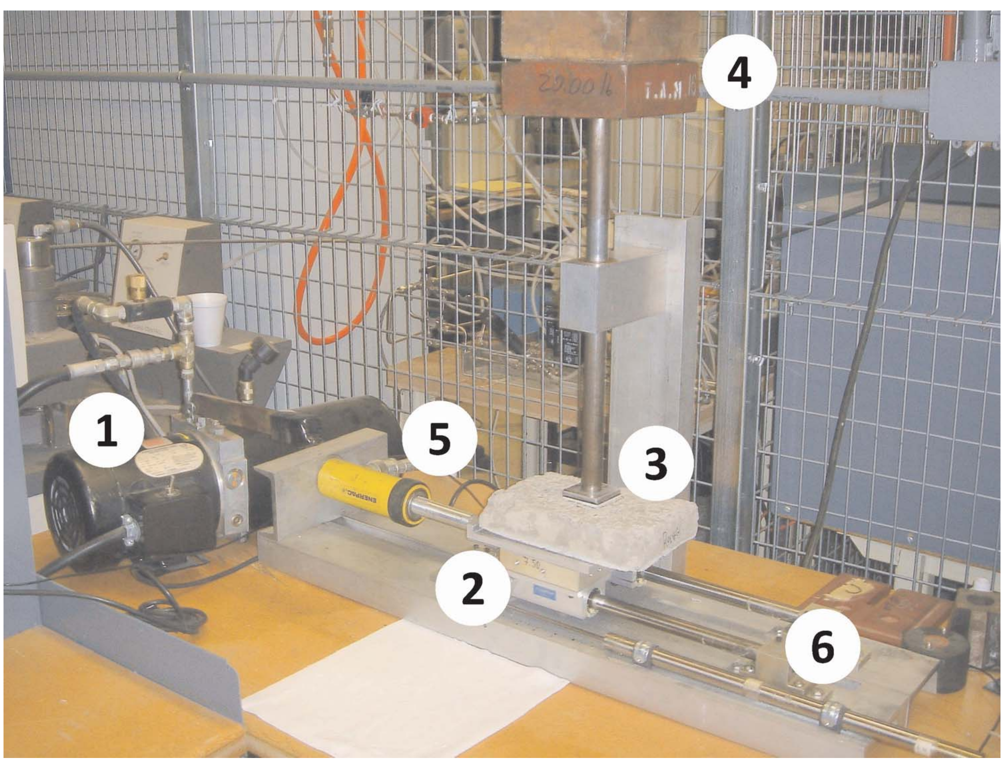

Figure 4: Apparatus used to measure the friction coefficient. 1: pump, 2: carriage, 3: sample holder, 4: weight, 5: load cell, 6: displacement transduced.

Tables

Manufacturer	Identification	Base	Coating	Weave	Tens. Strength Warp/Fill [N/mm]
Ferrari	Precontraint 1002 Formula S	Woven polyester	PVDF coating both sides	Plain weave	77 / 76
Seaman Corp.	Style 7150 PFF	Woven Nylon	Polyether Polyurethane both sides	Plain weave	131 / 131
Warwick Mills	4x4 Vectran	Woven Vectran	Urethane polymer both sides	Basket weave	406 / 450
Warwick Mills	2x2 Vectran	Woven Vectran	Urethane Polymer one side	Plain weave	400 / 400

Table 1: Fabric materials considered for the design.

	specimen	Fill direction R [N/mm]	Warp direction R [N/mm]
	1	443.246	387.030
	2	444.122	418.203
	3	468.815	397.188
	4	444.647	437.467
	5	447.799	363.038
	6	480.723	418.203
	7	441.495	437.292
	8	444.122	397.188
	9	412.599	405.419
	10	467.939	401.040
mean [N/mm]	$\bar{R}$	449.551	406.207
mle shape	$\hat{\lambda}$	458.141	416.264
mle scale	$\hat{k}$	27.080	21.441
data points	n	10	10
coverage	f	0.95	0.95
confidence	q	0.95	0.95
from table [14]	$V_{p,q,n}$	8.751	8.751
basis value	$x_{p,q}$	370.668	318.532
resist factor	φ_R	0.825	0.784

Table 2: Tensile strength of urethane impregnated/coated, 4x4 basket weave, Vectran 1500 denier.

		Concrete			
		Smooth		Rough	
		Dry	Wet	Dry	Wet
specimen		μ	μ	μ	μ
1		0.668	0.541	0.616	0.55
2		0.67	0.546	0.668	0.54
3		0.74	0.628	0.619	0.522
4		0.721	0.692	0.641	0.552
5		0.751	0.679	0.595	0.533
mean	$\bar{\mu}$	0.71	0.617	0.628	0.539
mean ln	x_{LN}	-0.343	-0.488	-0.466	-0.617
standard dev ln	s_{LN}	0.055	0.117	0.044	0.023
data points	n	5	5	5	5
coverage	f	0.95	0.95	0.95	0.95
confidence	q	0.95	0.95	0.95	0.95
from table [14]	$k_{p,q}$	4.203	4.203	4.203	4.203
basis value	$x_{p,q}$	0.563	0.375	0.522	0.49
resist factor	φ_μ	0.793	0.608	0.831	0.909

Table 3: Friction coefficient between concrete and urethane coated Vectran (1500 denier, 4x4 basket weave), applied pressure 239 kPa, and velocity 4.2 cm/s.

Variable	Description	Distribution	Parameter 1	Parameter 2
r_1	Test Pipe Radius	Deterministic	610 mm	-
r_2	Plug Hemisphere Radius	Deterministic	635 mm	-
L_c	Cyl. Portion Length	Deterministic	700 mm	-
p_i	Inflation Pressure	Normal	Mean=0.310 MPa	COV=0.10
p_e	Upstream Pressure	Normal	Mean=0.207 MPa	COV=0.05
R	Fabric Strength (Warp)	Weibull	λ=416.264 N/mm	k=21.441
μ	Fric. Coeff. (Smooth-Wet)	Log-Normal	Mean LN=-0.488	STDEV LN=0.117

Table 4: Variables definition for reliability analysis.

	Importance Factor for p_i (Eq. 23)		
$g_1 = p_i - p_e$	1	1.2	1.5
Mean p_e [KPa]	207.00	207.00	207.00
Mean p_i [KPa]	256.85	308.22	385.28
Std. dev. p_e [KPa]	10.35	10.35	10.35
Std. dev. p_i [KPa]	25.69	30.82	38.53
Reliability $\Re_{g1}$	0.964070	0.999065	0.999996
Probability of failure P_{Fg1}	0.035930	0.000935	0.000004

Table 5: Influence of importance factor on failure equation g_1.

	Importance Factor for p_i (Eq. 23)		
$g_2 = 2L_c p_i \mu - p_e r$	1	1.2	1.5
Mean p_i [KPa]	256.85	308.22	385.28
Std. dev. p_i [KPa]	25.69	30.82	38.53
Length L_c [mm] (Eq. 25)	813	677	542
Reliability $\Re_{g2}$	0.999978	0.999978	0.999978
Probability of failure P_{Fg2}	0.000022	0.000022	0.000022

Table 6: Influence of importance factor on failure equation g_2.

	Importance Factor for p_i (Eq. 23)		
$g_3 = R - p_i r/2$	1	1.2	1.5
Mean p_i [KPa]	256.85	308.22	385.28
Std. dev. p_i [KPa]	25.69	30.82	38.53
Stress [KPa] (Eq. 24)	117.33	140.80	176.00
Reliability $\Re_{g3}$	1.00	1.00	1.00
Probability of failure P_{Fg3}	$4.446\ 10^{-15}$	$2.492\ 10^{-13}$	$2.579\ 10^{-11}$

Table 7: Influence of importance factor on failure equation g_3.

		Importance Factor for p_i		
Limits for the whole system		1	1.2	1.5
Probability of failure	Lower limit	0.035930	0.000935	0.000022
	Upper limit	0.035952	0.000957	0.000026
Reliability	Upper limit	0.964070	0.999065	0.999978
	Lower limit	0.964048	0.999043	0.999974

Table 8: Limits of probability of failure and reliability of the system as function of the importance factor.

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