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# DETERMINATION OF BASIS VALUES FROM EXPERIMENTAL DATA FOR FABRICS AND COMPOSITES

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## ABSTRACT

Level III reliability analysis is proposed to deal with different statistical distributions for the various loads and resistances that affect the design of structural members and systems constructed of reinforced composite materials, textile soft goods, and other novel materials for which experimental data is scarce and onerous to obtain. The analysis is simplified in terms of basis values by treating independently each random variable in the design equation. Following this approach, the designer uses a single value (i.e. a basis-value) for each random variable in the design equation or equations while maintaining key characteristics of level III methodology. This is achieved by employing an appropriate distribution function for each random variable along with their first and second moments. A methodology to calculate basis-values other than A- and B-basis is presented in this work for the Normal, Log-Normal and Weibull distributions. A rationale for this approach and the need to expand the coverage to cases other than A- and B-basis is provided also.

## 1. INTRODUCTION

The use of advanced composite and textile materials is growing at a steady pace. New applications and materials are being developed every day. In some fields where these materials are used, design codes for composite materials exist. For example in the aeronautical industry, the MIL Handbook 17 Volume 1 Chapter 8 [16] specifies the procedures for calculating statistically based material properties. In many other areas there are no statistical codes for design or even there are no codes at all.

Fiber reinforced materials (e.g. composite materials, fabrics, etc.) present higher variability in their mechanical properties compared, for example, with homogeneous materials like steel or aluminum. The root cause of the higher variability can be traced back to the intrinsic variability of each constituent, surface phenomena between components, fabrication processes and even testing.

### 1.1 Design Methodologies

The most basic design method is to consider that both the actions (loads) and the resistance (material strength, section properties, etc.) are deterministic and a global safety factor is applied. This is known as Allowable Stress Design.

A more refined approach is to consider the variability in the actions and/or resistance, independently. In this approach characteristic or basis values for the actions and resistances are defined independently and the rest of the variables in the failure equation are considered deterministic.

Lastly, one could consider that an arbitrary number of the variables involved in the failure equation are non-deterministic and the reliability is assessed taking into account the probabilistic distribution and parameters for each random variable. If two or more random variables are not independent, the covariances should be taken into account.

### 1.2 Stochastic Models for Fiber Bundle Strength

The tensile strength of materials reinforced with long fibers can be analyzed using the concept of a fiber bundle material. The basic assumption is that the system works in parallel and that when a fiber breaks; it is no longer able to bear any load. Therefore the load must be redistributed among the surviving fibers [8, §9.1.3]. In the case of composite materials or wire rope, upon breakage of a fiber the load sharing takes place among the nearest fibers (local load-sharing). For composite materials the binding effect is due to matrix while for the wire rope is due to friction [10].

For materials reinforced with long fibers, if the strength of each individual fiber is assumed to be independent and to follow a Weibull distribution, it can be proved that the strength of the bundle is very well approximated with a Weibull distribution as well [3,10,11]. The parameters of the latter distribution will be different than the ones for the distribution of single fibers. For example, the shape parameter will be higher for the bundle, so that the variability of the strength will be lower for the composite that the single fibers.

The Weibull distribution for the single fibers is chosen because represents the response of a chain made of an ideal brittle material [8, §9.1.1] (weakest-link system). A further refined step would be to consider a bi-modal Weibull distribution, as is suggested in [1, §2.1], to include both intrinsic and extrinsic defects.

The Weibull model for composites is widely accepted and there is a strong preference for it, even if other models seem to do a better job fitting the data [16, §8.3.4].

## 2. BASIS VALUES

Resistance basis values are used in design to account for the variability of the material properties, while load basis values account for the variability of the loads. Basis values are one-sided tolerance intervals which contain a fixed proportion of the population $f$, also called coverage. In the case of resistances, only the lower tail of the distribution concerns. Because tolerance intervals are based on a sample $x$ containing only $n$ data points out of the entire population, the former assessment can be made only with a limited level of confidence $q$. As the number of samples $n$ tends to $\infty$ the basis value approaches the characteristic value.

There are two particular tolerance intervals that are widely used in the aerospace industry. These are the A- and B-basis tolerance intervals. The A-basis value of a random variable is such that 99 % of the data will fall above the basis value with a 95 % of confidence. The B-basis is less restrictive, demanding that only 90 % of the data has to be higher than the basis value, again with 95 % of confidence [16, §8.2.5.2].

The A-Basis is commonly used for catastrophic failure where there is no possible load redistribution. On the other hand, the B-Basis allowable is employed for structural members with redundant load paths where load redistribution is possible [5, §6.4].

In the aerospace industry, a serious failure is likely to be catastrophic so a higher reliability is expected but on the other hand, the mass of the structure is a major concern. For the sake of optimizing the mass while keeping the desired reliability, there are two sets of material allowables to be used depending on the nature of the consequences upon failure of the structural member in question.

As the use of fiber reinforced materials keeps spreading to other industries and applications, the designer might find that the A-Basis is too harsh while the B-Basis too mild. Furthermore, having to design using two different sets of allowables can be really cumbersome and error-prone. For some applications like wind turbine blades and marine structures it is common practice to use 95 % of coverage and 95 % of confidence in order to calculate the design values [12, 15, 17].

If experimental data is available, the mean value and standard deviation of the sample can be determined. Using the latter statistics and the sample size, the basis value can be calculated by the procedures described in this section. A goodness of fit test should be done to confirm that the chosen distribution adequately fits the data [16, §8.3.1].

### 2.1 Basis Values for the Normal Distribution

The basis value $x_{p,q}$ with coverage $f = 1 - p$ and confidence $q$ for a normal random variable $x$ with mean value and variance, both estimated by the sample mean $\bar{x}$ and sample standard deviation $s$, can be calculated as,

$$x_{p,q} = \bar{x} - k_{p,q}(n)\, s \tag{1}$$

In order to calculate $k_{p,q}(n)$, the non-central t-distribution with $\nu = n - 1$ degrees of freedom and non-centrality parameter $\lambda = \Phi^{-1}(f)\sqrt{n}$ is used, where $\Phi^{-1}$ is the inverse cumulative distribution function (icdf) for the standard normal. The coefficient $k_{p,q}(n)$ can be calculated as described in [2, 8, §3.3].

The different values of $k$ can be tabulated for a certain coverage and confidence as a function of the sample size $n$. Then, the basis value is calculated for any given sample mean and sample variance using (1).

Due to its widespread use in the aerospace industry, the values of $k$ have been tabulated for the A- and B-basis [16, Tables 8.5.11 and 8.5.10]. For values of $f$=0.95 and $q$=0.95 it is possible to build Table 1 with the value of $k$ as a function of $n$.

Table 1. One-sided basis tolerance limit factors, k, for the normal distribution with 95 % coverage and 95 % confidence

| **n** | **k** |
|---|---|
| 2 | 26.260 |
| 3 | 7.656 |
| 4 | 5.144 |
| 5 | 4.203 |
| 6 | 3.708 |
| 7 | 3.340 |
| 8 | 3.187 |
| 9 | 3.031 |
| 10 | 2.911 |
| 15 | 2.566 |
| 20 | 2.396 |
| 30 | 2.220 |
| 50 | 2.065 |
| 75 | 1.976 |
| ∞ | 1.645 |

In certain cases, where a particular material is made in large quantities following a very standardized process, the confidence on the standard deviation $s$ is very large and therefore it can be assumed to be known. Under this assumption, (1) is simplified to

$$k_{p,q}(\mathrm{n})\big|_{\sigma} = \Phi^{-1}(f) + \Phi^{-1}(q)/\sqrt{n} \qquad [2]$$

where the notation $(\cdot)|_{\sigma}$ is used to emphasize that $\sigma = s$.

In some codes, regulations or guidelines the statistical process to handle the variability of the material properties is simplified by using (2) instead of Table 1. The value of $k$ will be different depending on which equation is used, especially for small sample sizes. For example, in [15], a minimum of 5 samples is required. For the Normal distribution, with $n$ = 5 and assuming that the standard deviation is known, $k_{0.05,0.95}\big|_{\sigma}(5) = 1.645 + \frac{1.645}{\sqrt{5}} = 2.380$ (using (2)), yet from Table 1 for $k_{p,q}(5) = 4.203$. Therefore the basis value calculated using (1) will be different, depending on which equation was used to calculate $k$. It could be said that this is a simplified model and a safety factor can be applied to cover this inaccuracy, but by doing this the reliability will depend on the number of samples.

## 2.2 Basis Values for the Log-Normal Distribution

The Log-Normal Distribution assigns zero probability for negative values. This property is useful when, for some physical reason, the values of a certain random variable are known to be strictly positive. For example, the strength of a material cannot be negative.

The procedure to calculate basis values for a Log-Normal distributed variable is similar to the case of the Normal distribution. The difference is that the calculations must be done taking logarithms on the data. After the basis value is calculated on the transformed values, it must be transformed back to the original space applying the exponential function. So, the basis values is

$$x_{p,q} = exp\left[\bar{x}_{LN} - k_{p,q}(n)\, s_{LN}\right] \quad [3]$$

where $\bar{x}_{LN}$ is the sample mean in log space, $s_{LN}$ is the sample variance in log space and $k_{p,q}(n)$ is the same as in (1).

### 2.3 Basis Values for the Two-Parameter Weibull Distribution

The procedure to calculate a one-sided tolerance limit of a random variable *w* which is assumed to follow a two-parameter ($k$, $\lambda$) Weibull distribution, is explained in [7, Chapter 4] and with more detail in [6]. The shape parameter is $k$ and the scale parameter is $\lambda$. The drawback of the method presented in [6, 7] is that the results cannot be tabulated for an arbitrary sample characterized by Maximum Likelihood Estimates (m.l.e.'s), $\hat{k}, \hat{\lambda}$, for $k$ and $\lambda$, respectively.

The determination of the m.l.e.'s is done solving [7, Eqn. (4.1.8)]. Since it cannot be solved analytically, a numerical technique must be used. A FORTRAN subroutine is given in [16] or the MATLAB® command *mle* can be used.

The equivalent extreme value distribution with parameters $u$, $b$ is used for the derivation instead of the Weibull distribution with parameters $k$, $\lambda$. Therefore, all the observations $x_i$ must be converted to extreme value form,

$$w_i = ln(x_i) \quad [4]$$

The following step is to calculate the m.l.e.'s, $\hat{u}, \hat{b}$ for the extreme value distribution. Then, the relationship between $\hat{u}, \hat{b}$ and the m.l.e.'s $\hat{k}, \hat{\lambda}$ for the Weibull distribution is given by

$$\hat{k} = exp(\hat{u})$$
$$\hat{\lambda} = \hat{b}^{-1} \quad [5]$$

Where $(\hat{\cdot})$ indicates that the value is a m.l.e. and not the actual parameter.

Next, the set ***a*** of ancillary statistics are calculated as

$$a_i = \frac{w_i - \hat{u}}{\hat{b}} \quad [6]$$

The following equation must be solved for the basis value in extreme value form $t_{p,q}$

$$Pr\left(Z_p \leq t_{p,q}\big|_{\boldsymbol{a}}\right) = 1 - q \quad [7]$$

where the conditional distribution $Z_p$ given $\boldsymbol{a}$ can be found in [7, Eqn. 4.1.18] and $Pr(\cdot)$ is the probability.

The basis value in extreme value form is calculated as,

$$w_{p,q} = \hat{u} - t_{p,q}\,\hat{b} \tag{8}$$

Lastly, the basis value $w_{p,q}$ calculated in the previous step is in extreme value form and it should be converted to the original Weibull distribution,

$$x_{p,q} = exp\left(w_{p,q}\right) \tag{9}$$

This procedure has been developed in MATLAB and it is included in [2].

### *2.4 Weibull Basis Value Calculation using Tables for V*

Although the procedure explained in the previous section is enough for calculating the basis values, for design purposes the whole procedure might seem too complex. Part of the labor can be done once, for defined values of $p$ and $q$ and tabulated for different number of samples ($n$).

This has been done for the A and B basis in [16, Tables 8.5.8 and 8.5.9] for $n \geq 10$. As mentioned before, the aerospace industry can afford to test a large number of samples for each material (sometimes about 4000 coupons [9]), so it is reasonable that results from at least 10 samples will be available. On the other hand, other applications where composite materials are used, such extended testing is not feasible. The values of Table 2 for $n < 10$ should be used with caution. In [14] it is suggested that for very small samples, m.l.e.'s may not be very precise.

Table 2. One-sided basis tolerance limit factors, V for the Weibull distribution with 95 % coverage and 95 % confidence

| **n** | **V** |
|---|---|
| 2 | 98.081 |
| 3 | 17.884 |
| 4 | 16.748 |
| 5 | 13.333 |
| 6 | 11.558 |
| 7 | 10.463 |
| 8 | 9.715 |
| 9 | 9.169 |
| 10 | 8.751 |
| 15 | 7.566 |
| 20 | 6.995 |
| 50 | 5.915 |
| 75 | 5.636 |

The $V$ coefficient plays a similar role as $k$ for the normal and log-normal distributions. The most important difference is that in the calculation of the basis values the mean value and standard deviation are not used. The calculation of the basis values is done using the m.l.e's for the Weibull distribution (shape parameter $\hat{k}$ and scale parameter $\hat{\lambda}$) and the corresponding $V$ coefficient in (10).

$$x_{p,q} = \hat{\lambda}[-ln(1-p)]^{1/\hat{k}} exp\left(\frac{-V_{p,q,n}}{\hat{k}\sqrt{n}}\right) \qquad [10]$$

The procedure used to calculate $V$ for any coverage, confidence and number of samples is described in detail in [2].

## 3. EXAMPLES BASED ON EXPERIMENTAL RESULTS

Examples employing the concepts described in the present work will be applied to the design of an inflatable structure. The purpose of this structure is to seal a cylindrical conduit in case of flood. The structure is composed of a cylindrical body with spherical caps.

### 3.1 Short Term Strength

The calculation of the basis value for the short term strength of a Vectran reinforced 1500d, Urethane coated fabric, un-cut, along the fill direction is summarized in Table 3. The experimental data is taken from [13]. The data is assumed to follow a Weibull distribution, based on the reasons given in section 1.2. Furthermore, the Anderson-Darling test was performed on the data and the observed significance level (OSL) was greater than 0.25. The MIL-Handbook-17-1F [16] suggests that if OSL > 0.05, the Weibull distribution should be adopted. The basis value for a coverage of 95 % and a confidence of 95 % are calculated using equation (10) and the values of *V* from Table 2.

Table 3. Data and calculation of short term strength basis value for Urethane/Vectran 1500d un-cut samples in the fill direction with 95 % coverage and 95 % confidence.

| | **Specimen** | **Strength F [N/mm]** |
|---|---|---|
| | 1 | 363.493 |
| | 2 | 361.682 |
| | 3 | 372.050 |
| | 4 | 381.196 |
| Mean | Mean(F) | 369.605 |
| Scale Parameter (m.l.e.) | $\hat{\lambda}$ | 373.5425 |
| Shape Parameter (m.l.e.) | $\hat{k}$ | 49.2122 |
| Sample Size | n | 4 |
| Coverage | f | 0.95 |
| Confidence | q | 0.95 |
| From Table 2 | $V_{p,q,n}$ | 16.748 |
| Basis Value Eq. (10) | $x_{p,q}$ | 296.6389 |

### 3.2 Friction

For the calculation of the allowable value for the friction coefficient it will be assumed that random variable follows a log-normal distribution. The log-normal distribution assigns zero probability for negative values. In this example, the friction coefficient cannot be negative. The coverage and confidence chosen are 95 %.

Table 4. Data and calculation of friction coefficient basis value with 95 % coverage and 95 % confidence.

| | **Specimen** | **Friction Coefficient μ** |
|---|---|---|
| | 1 | 0.658 |
| | 2 | 0.678 |
| | 3 | 0.725 |
| | 4 | 0.719 |
| | 5 | 0.715 |
| | 6 | 0.737 |
| | 7 | 0.727 |
| | 8 | 0.708 |
| | 9 | 0.719 |
| | 10 | 0.709 |
| Mean(ln(μ)) | $\bar{x}_{LN}$ | −0.344 |
| Standard Dev(ln(μ)) | $s_{LN}$ | 0.035 |
| Sample Size | n | 10 |
| Coverage | f | 0.95 |
| Confidence | q | 0.95 |
| From Table 1 | $k_{p,q,n}$ | 2.911 |
| Basis Value Eq. (3) | $x_{p,q}$ | 0.641 |

### 3.3 Long Term Strength

The data included in Table 5 from [13], shows a set of time to failure and load at failure data points for Vectran fabric with a 2x2 weave. The long term strength (28 days) was an important parameter for the design of the inflatable structure. To estimate the parameters of the strength distribution at 28 days, it was assumed that the data fits a Bivariate Normal Distribution. The procedure used is explained in the following steps,

1. Calculate $\ln(t_i)$ for all the times to failure. The logarithm of the time to break does a better job fitting the available experimental data.
2. Calculate the mean values and standard deviation for the natural logarithms of the times to failure **t** and strengths ***R***.
3. Find the best fit for the linear correlation factor ρ for the data.
4. Calculate the conditional distribution of the Bivariate Normal Distribution with parameters $\mu_{t_{LN}}$, $\sigma_{t_{LN}}$, $\mu_R$, $\sigma_R$ and ρ, at 28 days using the conditional moments.

Table 5. Long-term Strength and Time to Failure for Vectran 2x2 Fabric

| **R[N/mm]** | **t[min]** | **R[N/mm]** | **t[min]** | **R[N/mm]** | **t[min]** | **R[N/mm]** | **t[min]** |
|---|---|---|---|---|---|---|---|
| 350.4 | 0.52 | 324.1 | 0.87 | 306.6 | 7.33 | 284.7 | 1280 |
| 350.4 | 0.75 | 324.1 | 6.28 | 306.6 | 120.88 | 284.7 | 1278 |
| 350.4 | 0.9 | 324.1 | 6.47 | 297.8 | 38.07 | 284.7 | 724 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 350.4 | 0.63 | 315.4 | 4.42 | 297.8 | 140.98 | 284.7 | 169 |
| 341.6 | 0.69 | 315.4 | 4.62 | 297.8 | 117.87 | 284.7 | 787 |
| 341.6 | 2.22 | 315.4 | 4.16 | 297.8 | 81.58 | 280.3 | 632.38 |
| 341.6 | 1.26 | 315.4 | 1.53 | 297.8 | 15.22 | 280.3 | 3132.47 |
| 341.6 | 1.07 | 315.4 | 0.61 | 297.8 | 90.52 | 280.3 | 854 |
| 332.9 | 4.28 | 306.6 | 6.67 | 289.1 | 53.08 | 280.3 | 48 |
| 332.9 | 1.15 | 306.6 | 8.17 | 289.1 | 91.5 | 280.3 | 748 |
| 332.9 | 0.88 | 306.6 | 65.2 | 289.1 | 51.65 | 280.3 | 293 |
| 332.9 | 5.18 | 306.6 | 104.67 | 289.1 | 284.97 | 280.3 | 1440 |
| 324.1 | 6.42 | 306.6 | 32.95 | 289.1 | 127.27 | | |
| 324.1 | 1.13 | 306.6 | 43.53 | 289.1 | 59.95 | | |

Table 6. Calculation of long-term strength basis value for Vectran 2x2 fabric with 95 % coverage and 95 % confidence.

| | | |
|---|---|---|
| Log Time to failure mean | $\mu_{t_{LN}}$ | 3.1208 |
| Log Time to failure standard dev | $\sigma_{t_{LN}}$ | 2.5412 |
| Strength mean | $\mu_R$ | 308.4556 |
| Strength standard dev | $\sigma_R$ | 22.4082 |
| Correlation | $\rho$ | -0.9052 |
| Conditional strength mean @ 28 days | $\mu_{R\ @\ 28\ days}$ | 248.7210 |
| Conditional strength std dev @ 28 days | $\sigma_{R\ @28\ days}$ | 9.5250 |
| Sample Size | n | 54 |
| Coverage | f | 0.95 |
| Confidence | q | 0.95 |
| From Table 1 | $k_{p,q,n}$ | 2.0463 |
| Basis Value Eq. (1) | $x_{p,q}$ | 229.2300 |

# 4. CONCLUSIONS

The underlying physical reason supporting the use of a Weibull distribution for the strength in the fiber direction of a fabric or composite reinforced with long fibers was outlined in the introduction of this work. The procedure to compute statistically based material values (basis values) $x_{p,q}$ is very simple. When the data follows a Weibull distribution, a procedure to compute maximum likelihood estimates is needed, but such a procedure is readily available both in commercial and free software packages. The procedures to calculate the coefficients $k_{p,q,n}$ and $V_{p,q,n}$ that simplify the calculation of basis values for Normal and Weibull distributed data, respectively, are somewhat complex, but the results have been tabulated in Tables 1 and 2 even for very small number of data points $n$. Log-normally distributed data is handled using the same table used for Normal data. The multivariate Normal distribution was used to calculate the statistically based strength, taking into account the loading duration. The tabulated values for 0.95 coverage and 0.95 confidence, should be very useful in many applications when a unique basis value is desired and the traditional A- and B-basis prove to be too strict and too lax, respectively.

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