Beyond Plain Weave Fabrics - I. Geometrical Model

A. Adumitroaie and E. J. Barbero¹ Mechanical and Aerospace Engineering, West Virginia University, Morgantown, WV 26506-6106, USA

Abstract

In response to the large variety of weaving styles offered by the textile industry, a new general approach for the geometrical modeling of 2D biaxial orthogonal woven fabric reinforcements for composite materials is proposed here. New geometrical parameters are introduced in order to describe general families of twill and satin woven patterns, and a new classification of woven fabrics is proposed based on these parameters. Generation of the 3D internal geometry of the woven fabric families is achieved based on new geometrical functions that consider the actual configuration of the composite material in all its complexity. The proposed geometrical model is intended as the foundation for further analytical or numerical modeling of the mechanical properties of the composite materials reinforced with these fabrics.

Keywords

Textile, Fabric, Composite, Plain, Twill, Satin, Weave, Geometrical model.

1 Introduction

Textile reinforcements are widely used as an alternative to traditional unidirectional reinforcements for structural applications of composite materials in aerospace, automotive, civil, marine and other industries. The most commonly used textile structures are woven, braided, knitted, and non-woven fabrics. A comprehensive classification of textile reinforcements is offered in [1,2].

The present study focuses on 2D woven reinforcements, among which plain, twill, and satin weave are distinct categories. 2D woven fabrics are obtained by interlacing two sets of tows, strands, or yarns in the weaving machine. The tows running along the weaving direction are called warp, while the ones running transverse to the weaving direction are called fill or weft. This type of fabrics are labeled 2D because they provide only in-plane reinforcing properties at the laminate level; similar to traditional unidirectional ply laminates, there are no through the thickness reinforcing elements. 2D weaves are biaxial (i.e., the tows are aligned along only two directions, which are the fill and warp directions) and orthogonal (i.e., the fill and warp tows are laid down at 90° with respect with each other). Even if the fabric reinforcement features in-plane 2D reinforcing properties, it still has a 3D internal architecture due to its waviness. Waviness results in complex stress-strain fields, which usually translates in reduction of the material properties and the generation of new failure modes compared to unidirectional reinforcements.

Most of the literature deals with plain weave fabric reinforcements [3-14], or with particular cases of twill/satin weaves by building individual models for each of them [15-21]. Recently, efforts have been made to construct general models, applicable to classes of weaving styles rather than to particular cases [22-24].

The capacity of a model to predict damage initiation and strength is directly related to the quality of the geometrical description of the reinforcing architecture. Simplifying assumptions made on the internal

¹Corresponding author. The final publication is available at http://dx.doi.org/10.1016/j.compstruct.2010.11.014

geometry result in loss of accuracy in the resulting stress-strain field. A certain degree of geometrical simplification may be insignificant if the model is dedicated to evaluating the thermo-elastic properties of the material, but becomes significant when the model is dedicated to evaluate progressive damage.

If the model is based on a geometry description considering the tow undulation in both fill and warp directions, as in [3-6, 8-10, 14, 18-26], it is regarded as a 3D model. Opposite, a model may involve simplifications (e.g., considering the tow undulation in only one of the fill/warp directions [11, 12, 15, 16, 27]) that reduce the model to a 2D model. Furthermore, various assumptions can be identified in different models in the literature regarding the tows cross-sectional shape, tow undulated shape, and the gap between adjacent tows. Higher accuracy in modeling the geometry of the reinforcing structure results in higher accuracy of the model output, but higher model setup effort, computational effort, and severely limited variety of fabric architectures, often limited to plain weave only. The most accurate geometrical descriptions are usually used in numerical (FEA) models, while analytical models rely on various geometrical simplifications.

Elastic properties of plain and satin weave reinforced composite materials are predicted in [15,27] using mosaic, crimp and bridging models. The mosaic model does not consider the undulation of any fill/warp tows, and the material is considered as an assemblage of cross-ply type elements. The crimp model, also called the fiber undulation model, considers the tow waviness along one direction only. The bridging model is a combination of the previous two models, being dedicated to more accurate evaluation of the elastic material properties of satin weaves. The bridging model is based on repetitive unit cell (RUC) technique, and the selected RUC applies to only 8-harness satin, with the results being latter applied to other satin types.

Improvement of the fiber undulation model is done in [11, 12] for plain weave only. Additional aspects of the material architecture are considered, as the phase shift in between different layers of the laminate, but without considering the plies nesting triggered by the phase shift. The model features the same 2D geometrical representation as the fiber undulation model, considering the tow undulation along one direction only. A more advanced 3D geometrical model for plain weave fabrics is developed in [12].

A comprehensive analysis of composite materials based on plain weave fabrics is made in [3, 4, 6, 8-10]. In [3, 6] a 3D geometrical model considering the fiber undulation on both fill and warp directions, as well as the gap in between individual tows, and the possibility for different cross-sectional parameters of fill and warp tows is presented. A very detailed analysis of the possibilities of geometrical modeling of the 3D architecture of the plain weave reinforcement is presented in [8], clearly pointing out the possible inaccuracies induced by different geometrical representations. The possible phase shift in between different plies of the laminate, and the nesting effect triggered by phase shift is accounted for.

Two specific cases of satin weave (5– and 8–harness) are modeled in [18,19]. Particular cases of twill weave, the twill family "1–under/n–over" featuring the thinnest diagonal rib and the twill pattern "2–under/2–over", are presented as separate models in [20,21].

Individual cases of plain, 5-harness, and 8-harness weaves are treated in [17], based on a not very accurate 3D geometrical model that uses rectangular cross-sections for the yarns. The geometrical models for these three cases are regarded as separate problems, without generalizing the process to generate the geometry.

An attempt towards generalizing the analysis of 2D weaves and braids is made in [23, 28], where a set of four parameters is assigned to each fill and warp tow in order to describe the inter-weaving position (top or bottom) and length of tow segments inside RUC. The goal is to describe the undulation of fill/warp tows at any location of the RUC, based on the four-parameter sets assigned to each individual fill/warp tows. However, the model in [23, 28] does not have the ability to automatically generate weaving families, because individual sets of parameters must be assigned to individual fill/warp tows for individual weaving styles. Moreover, the value of each parameter in the set is also a function of the weaving style. Because of these drawbacks, only braiding cases are treated in [23, 28] for fixed interlacing pattern and only varying the braiding angle.

Another attempt to systematically treat the problem of woven fabric families is done in [22]. The dependence of final composite material properties on the geometrical configuration of the reinforcement are stated, and the model is meant to deal with the challenge of the large variety of woven fabric architectures, being able to accommodate tow orientations other than orthogonal. Geometrical functions are provided for individual segments of the undulated tows, but no methodology is provided to generate general weaving patterns. Because of this, the analysis is performed on the particular cases of plain weave and "1–under/2–over" twill weave, which are actually described by separate geometrical models.

The need for accurate and precise geometrical model is the objective of some of the research studies that focus exclusively on the geometrical description of the architecture of the fabric reinforcement [24–26,29,30]. In [26] some of the inaccuracies of the previous models are identified and corrected for the particular case of plain weave fabric. In [29] a new methodology of building generic geometrical models is launched, even if accurate details are not considered (e.g. the tow cross-section is rectangular, and the fill and warp tow are identical). The method consists of generating libraries of building blocks, including all the possible combinations of fill/warp stacking and undulation, and assembling these building blocks (similar to a puzzle) to generate a certain woven pattern. In this way, the final outcome is not a specific description of a specific woven fabric, but it is rather a collection of small pieces that can generate any weaving style, if assembled in the right way. The assemblage of the blocks is a separate aspect of the problem, and it is done manually in [29], by laying down in the right way individual building blocks, piece by piece, filling separate tows of the RUC, which is tedious and prone to mistakes. The final result is a 3D geometrical model, and the technique is pretty much similar to CAD modeling designated to subsequent meshing for FE analysis.

The geometrical modeling technique in [29] is further developed [24, 25] by increasing the accuracy and generality of the building blocks. Normally, the higher the number of the aspects considered (e.g. different geometrical parameters for fill and warp tows, or considering the hybridization for material properties and/or geometrical parameters of the fill/warp tows), the wider the resulting library of the building blocks, and the more complicated the assemblage of the blocks into a certain fabric type. This is done in [25], where a meshing procedure is also proposed, which will be further used in [30] in order to calculate the 3D elastic properties of the material, based on either iso-strain or iso-stress assumption at the RUC level. The inherent complexity of assembling building blocks is obvious in that the only case illustrated in [30] is that of a plain weave. This problem is addressed in [24], where, besides an accurate geometry description, the assembly of the geometrical blocks is automatically done based on a mapping of the RUC. According with this map, which becomes an input of the geometrical model generation, individual blocks are assigned to individual locations of the RUC, and the 3D representation of a certain weaving style is obtained. In addition to the mapping matrices, two vectors describing the centroidal coordinates of the individual blocks along the fill and warp directions are also required as input data. The amount of input data is large, and categories of weaves (e.g. a whole set of satins or a whole set of twills) cannot be obtained automatically. The geometrical model generation is exemplified by the particular cases of plain weave and 4-, 5-, 8-harness satin weave. Is has to be noted that the problem becomes even more complicated when tow width, thickness, and gap are different for fill and warp.

In view of the limitations of the available model generation techniques, the objective of the present work is to propose an analytical formulation capable of generating the 3D geometry of broad families of weaves, such as twill and satin, in terms of a few parameters.

2 Description and classification of weaving styles

Examples of 2D, biaxial, orthogonal woven fabrics are presented in Fig. 1.

The fill and warp tows are described by the following parameters (see Fig. 2):

- The width of fill a_f and warp a_w ,
- The tow thickness of the fill h_f and warp h_w , and
- The gap between two adjacent tows g_f and g_w .

The geometrical parameters of the tow play a role on the drapeability and snag resistance of the dry fabric, on the permeability of the fabric during resin infusion, and on the final properties of the fabric reinforced composite material, including the thermo-elastic properties, damage onset, and strength. Often a trade-off between manufacturability and performance is necessary, thus requiring the ability to predict the material performance for a number of candidate fabric weaves.

Fabrics can be woven in different styles and patterns (Fig. 1, 3), namely plain, twill, satin, and basket weaves, each of them featuring some advantages and disadvantages from the processing and manufacturing point of view, as well as having a different impact on the mechanical properties of the composite material.



Figure 1: Examples of 2D, biaxial, orthogonal weaving patterns: a) plain, b) twill, c) satin.

The plain weave (Fig. 1 a) is the most common weaving style, thus enjoying the highest attention from the research community. The fill and warp tows are interlaced in an "one under – one over" pattern that generates a chessboard pattern with equally exposed fill and warp tows on both faces of the fabric. Plain weave fabrics consolidate well, are easy to handle, are wear and snagging resistant, but they do not drape well to complex mold shapes, tending to wrinkle.

The twill weave (Fig. 1 b, Fig. 3 b and c) is characterized by continuous rib diagonals on the faces of the fabric. In the figures, the fill and warp tows are depicted horizontally and vertically, respectively. In a twill, the fill tows are interlaced in a pattern of "m under – n over", with at least one of the m, n > 1, and featuring a shift n_s (Fig. 4 a) in the fill direction of at most m, i.e., $n_s \leq m$, so that a diagonal rib is generated on each face of the fabric. Many twill weave styles can be designed, having different width of the diagonal ribs, or having a balanced (Fig. 2) or unbalanced (Fig. 3 b and c) fill and warp exposure on the faces of the fabric. The main characteristic of the twill weave is its improved drapeability as compared to plain weave, being at the same time prone to snagging when the harness is large and/or the diagonal ribs are thin.

Satin weaves are depicted on Figs. 1 c, 3 d, and 3 e, again with the fills horizontal and the warps vertical. A twill is generated by assigning values to the "m under – n over" parameters of fill interlacing in such a manner that no continuous diagonals can be identified on the faces of the fabric. This can be achieved by a shift $n_s > m$ (Fig. 4 b). Exposure of the tows on the faces of the fabric is unbalanced, dominated by either fill or warp on opposite faces. A satin provides the best drape, being at the same time the most prone to snagging due to lack of connectivity between adjacent, parallel tows.

Similar appearance to plain weave is featured by the basket weave, where the "one under – one over" interlacing pattern is replaced by a balanced "m under – m over" one, with a shift equal to m in between tow groups, with the particularity that, in this case, the shift is applied to a whole group of m adjacent tows rather that to individual tows. In this manner, the fabric face has a chessboard appearance, with each square composed by m individual tows, with m = 1 for the case of plain weave. The drapeability of the basket weave is improved in this way, compared to plain weave.

The repetitive unit cell (RUC) is used here for geometrical and mechanical analysis. There are multiple ways to select a RUC for the same fabric, using either one or the other face of the fabric, setting the horizontal axis along either fill or warp direction, and describing the fabric architecture as it develops on the positive or negative side of the axes. Once a RUC is selected, it becomes representative for the material, and the calculated properties are unique, regardless of the way the RUC was selected. However, a unique way of selecting the RUC is required for computer implementation of the model. In order to establish uniqueness, to be in accordance with the notation used in the present paper (i.e., the horizontal x direction of the coordinate system along the fill tow), and to assure consistence with the model formulation, the RUC of any weaving style has to be selected such that the origin of the coordinate system, corresponding to the left-bottom



Figure 2: Geometrical parameters of individual fill and warp tows of a 4/1/2 twill. Subscripts denote: f=fill, w=warp, r=resin.

corner of the RUC, is located at the beginning of an "under" segment of the fill (see Fig. 1, 2, 3).

In this way, an unique RUC can be selected, which is in agreement with the geometrical parameters introduced in this work in order to describe the weaving pattern in a manner amenable for computer implementation. These parameters are n_q, n_s, n_i (Fig. 4), defined as follows:

- The harness n_g is the dimension of the repetitive unit cell (RUC), expressed in number of tows (e.g., $n_g = 3$ in Fig. 1 b.)
- The shift n_s in the fill direction required to repeat the interlacing pattern on consecutive fill tows (look at fill f_2 in Fig. 4 a, and count the shift to the right required for fill f_3 to display the same interlacing pattern; i.e., $n_s = 1$.)
- The interlacing n_i is the number of warp tows over a fill tow in the interlacing region (e.g., $n_i = 2$ in Fig. 4 a)

Making use of these parameters, a new codification system for 2D biaxial orthogonal woven fabrics is proposed and used here. According to this system, any weaving style is described by the combination of the three parameters $n_g/n_s/n_i$, without the necessity of generating individual geometrical descriptions for particular cases.

3 Geometrical model of weaving patterns

A complete geometrical representation of the fabric structure, at any (x, y) location of the RUC, requires to know:

- The undulation along the fill and warp tows, $z_f(x, y)$ and $z_w(x, y)$ (see Fig. 5) and
- The cross-sectional shape of the tows, $e_f(x, y)$ and $e_w(x, y)$ (see Fig. 5.)



Figure 3: Twill and satin weaving patterns, having $n_g = 5$ and different values of n_s , n_i parameters: a) 5/1/1, b) 5/1/2, c) 5/1/3, d) 5/2/1, e) 5/3/1.

These functions are different for each type of weave, namely twill and satin. Therefore, they are given separately in Sections 3.1, 3.2, and 3.3.

Once the undulated shape and cross section of the tow are known at any (x, y) location of the RUC, all other quantities required by a mechanical model can be calculated in terms of the geometrical parameters $a_f, g_f, h_f, a_w, g_w, h_w$ of individual fill/warp tows (Fig. 2), as follows:

The top and bottom surfaces of the fill and warp tows

$$z_{f}^{top}(x,y) = z_{f}(x,y) + \frac{1}{2} e_{f}(x,y)$$

$$z_{f}^{bot}(x,y) = z_{f}(x,y) - \frac{1}{2} e_{f}(x,y)$$

$$z_{w}^{top}(x,y) = z_{w}(x,y) + \frac{1}{2} e_{w}(x,y)$$

$$z_{w}^{bot}(x,y) = z_{w}(x,y) - \frac{1}{2} e_{w}(x,y)$$
(1)

The fill and warp undulation angles

$$\theta_f(x,y) = \arctan\left(\frac{\partial}{\partial x} z_f(x,y)\right)$$

$$\theta_w(x,y) = \arctan\left(\frac{\partial}{\partial y} z_w(x,y)\right)$$
(2)



Figure 4: Geometrical parameters of the weaving pattern.

The length of a fill/warp tow inside the RUC

$$L_f = \int_0^{n_g \cdot w_w} \sqrt{1 + \left(\frac{d}{dx} z_f(x, y)\right)^2} dx$$
$$L_w = \int_0^{n_g \cdot w_f} \sqrt{1 + \left(\frac{d}{dy} z_w(x, y)\right)^2} dy \tag{3}$$

where $w_f = a_f + g_f$ and $w_w = a_w + g_w$. The crimp of fill/warp tows is defined as the percent fraction between the total length of the tow inside the RUC L_f , L_w , and the length of the RUC along the corresponding tow direction L'_f , L'_w , as follows

$$c_f = 100 \left(1 - \frac{L'_f}{L_f} \right) [\%]$$

$$c_w = 100 \left(1 - \frac{L'_w}{L_w} \right) [\%]$$
(4)

where L'_f, L'_w dimensions are calculated as $L'_f = n_g(a_w + g_w) = n_g w_w$ and $L'_w = n_g(a_f + g_f) = n_g w_f$. The individual fill/warp cross-sectional areas

$$A_f = \int_0^{w_f} e_f(x, y) dy$$
$$A_w = \int_0^{w_w} e_w(x, y) dx$$
(5)

The volume of a fill/warp tow inside RUC

$$v_f = A_f \ L_f$$
$$v_w = A_w \ L_w \tag{6}$$



Figure 5: Geometrical representation of the fabric reinforcing structure of a 4/1/2 twill weave.

The meso-scale volume fraction of fill and warp tows inside the RUC

$$V_{meso} = \frac{n_g(v_f + v_w)}{v_{RUC}} \tag{7}$$

The total volume of the RUC

$$v_{RUC} = n_g^2 w_f w_w h \tag{8}$$

These quantities can be used in a mechanical model of the fabric reinforced composite material in order to calculate the thermo-elastic constants, damage initiation, and strength. The input data for the geometrical model are:

- The individual fill/warp tows geometrical parameters $a_f, g_f, h_f, a_w, g_w, h_w$ and
- The parameters describing the weaving pattern n_g, n_s, n_i .

The 3D internal architecture of the reinforcement is accounted for by considering the tow undulations in both fill and warp directions as well as the lenticular shape of the tow cross-section, according with experimental photomicrographic observation of woven fabric reinforced composite materials [31, 32]. The proposed model allows for different input geometrical parameters of individual fill and warp tows (tow width, tow maximum thickness, or the gap between adjacent tows), as it might be the case as a result of engineering design or processing conditions.

The tow waviness $z_f(x, y), z_w(x, y)$, (Fig. 5) can be described by continuous undulated shapes, or by combinations between undulated and straight segments. Continuous sinusoidal undulation functions have been used in [3–10] for the plain weave case and in [20,21] for few particular cases of twill and satin weaves.

The tow cross-section $e_f(x, y)$, $e_w(x, y)$, (Fig. 5) can be described by circular, elliptical or trigonometrical shapes, with or without flattened segments. These functions are presented in Sections 3.1, 3.2, and 3.3 in order to describe complete families of twill and satin weaves with arbitrary values of weave parameters n_g , n_s , n_i and tow parameters a_f , g_f , h_f , a_w , g_w , h_w .

3.1 Twill weave family with $n_q \ge 2, n_s = 1, n_i = 1$

All twills have $n_q > 2$, with plain weave being a particular case at $n_q = 2$.

The geometrical functions $z_f(x, y)$, $e_f(x, y)$, $z_w(x, y)$, $e_w(x, y)$ presented here are meant to describe a class of twill weaves with arbitrary harness n_g , but having shift $n_s = 1$ and interlacing $n_i = 1$, as depicted in Fig. 1 a, and b. This twill family is characterized by the thinnest diagonal ribs, due to the minimum value of the interlacing $n_i = 1$. In the codification system proposed in here, this twill family is denoted as $n_g/1/1$.

This family is a particular case of the more general $n_g/1/n_i$ described in Section 3.2, but it is shown here because it affords certain simplifications that make the explanation easier, more didactic, and because it is of common use in industry.

The function to describe the undulation $z_f(x, y)$ (Fig. 5) of the fill tow is

$$\begin{cases} \text{if } (j-1)w_f + \frac{g_f}{2} \le y \le jw_f - \frac{g_f}{2} \\ z_f(x,y;j) = \\ \begin{pmatrix} -\frac{h_f}{2}sin\left[\frac{\pi x}{w_w} + (j-1)\pi\right] & \text{if } (j-\frac{3}{2})w_w \le x \le (j+\frac{1}{2})w_w \\ \frac{h_f}{2} & \text{if } (j+\frac{1}{2}-n_g)w_w < x < (j-\frac{3}{2})w_w \\ \text{or } (j+\frac{1}{2})w_w < x < (j-\frac{3}{2}+n_g)w_w \\ (-1)^{n_g+1} \times \\ \frac{h_f}{2}sin\left[\frac{\pi x}{w_w} + (j-1)\pi\right] & \text{if } (j-\frac{3}{2}-n_g)w_w \le x \le (j+\frac{1}{2}-n_g)w_w \\ \text{or } (j-\frac{3}{2}+n_g)w_w \le x \le (j+\frac{1}{2}+n_g)w_w \\ \end{cases}$$
(9)

$$\text{if } (j-1)w_f \le y < (j-1)w_f + \frac{g_f}{2} & \text{or } jw_f - \frac{g_f}{2} < y \le jw_f \\ z_f(x,y;j) = 0 \end{cases}$$

and the function to describe the undulation $z_w(x, y)$ of the warp tow is

$$\begin{cases} \text{if } (i-1)w_w + \frac{g_w}{2} \le x \le iw_w - \frac{g_w}{2} \\ z_w(x,y;i) = \\ \begin{pmatrix} \frac{h_w}{2}sin\left[\frac{\pi y}{w_f} + (i-1)\pi\right] & \text{if } (i-\frac{3}{2})w_f \le y \le (i+\frac{1}{2})w_f \\ -\frac{h_w}{2} & \text{if } (i+\frac{1}{2}-n_g)w_f < y < (i-\frac{3}{2})w_f \\ \text{or } (i+\frac{1}{2})w_f < y < (i-\frac{3}{2}+n_g)w_f \\ (-1)^{n_g} \times \\ \frac{h_w}{2}sin\left[\frac{\pi y}{w_f} + (i-1)\pi\right] & \text{if } (i-\frac{3}{2}-n_g)w_f \le y \le (i+\frac{1}{2}-n_g)w_f \\ \text{or } (i-\frac{3}{2}+n_g)w_f \le y \le (i+\frac{1}{2}+n_g)w_f \\ \end{cases} \end{cases}$$
(10)

$$\text{if } (i-1)w_w \le x < (i-1)w_w + \frac{g_w}{2} & \text{or } iw_w - \frac{g_w}{2} < x \le iw_w \\ z_w(x,y;i) = 0 \end{cases}$$

Note that two auxiliary parameters $i, j = 1 \cdots n_g$ (Fig. 3 a) refer to each individual tow in the RUC (see Fig. 3). This means that equations (9)–(10) are able to describe the fill and warp undulated shape one tow at a time. For example, the function $z_f(x, y; j)$ describes the waviness of the fill tow j inside the RUC, at any (x, y) location inside the tow, while the function $z_w(x, y; i)$ describes the waviness of the warp tow i inside the RUC. Having the waviness defined as a function of both x and y allows for *analytical* computation of all the quantities required for the analysis of textile reinforced composites.

In a similar way, making use of the i, j parameters to identify individual fill/warp tows inside the RUC, the cross-sectional thickness functions for the fill and warp tows are described by

$$e_{f}(y;j) =$$

$$\begin{cases} \left| h_{f}sin\left[\frac{\pi(-(j-1)w_{f}+y-g_{f}/2)}{a_{f}}\right] \right| & \text{if } (j-1)w_{f} + \frac{g_{f}}{2} \le y \le jw_{f} - \frac{g_{f}}{2} \\ 0 & \text{if } (j-1)w_{f} \le y < (j-1)w_{f} + \frac{g_{f}}{2} & \text{or } jw_{f} - \frac{g_{f}}{2} < y \le jw_{f} \end{cases}$$
(11)

and

$$e_{w}(x;i) =$$

$$\begin{cases} \left| h_{w}sin\left[\frac{\pi(-(i-1)w_{w}+x-g_{w}/2)}{a_{w}}\right] \right| & \text{if } (i-1)w_{w} + \frac{g_{w}}{2} \le x \le iw_{w} - \frac{g_{w}}{2} \\ 0 & \text{if } (i-1)w_{w} \le x < (i-1)w_{w} + \frac{g_{w}}{2} & \text{or } iw_{w} - \frac{g_{w}}{2} < x \le iw_{w} \end{cases}$$
(12)

Note in (11)–(12) that the cross-section of the fill is assumed to be constant along its length (x) and the cross-section of the warp is constant along its length (y).

The following procedure identifies the region of applicability (domain) of equations (9) - (12) in terms of the tow identifiers i, j, as follows:

when
$$(j-1) w_f \le y \le j w_f$$

 $z_f(x,y) = z_f(x,y;j)$, $e_f(y) = e_f(y;j)$
when $(i-1) w_w \le x \le i w_w$
 $z_w(x,y) = z_w(x,y;i)$, $e_w(x) = e_w(x;i)$ (13)

In this way, the $z_f(x, y), z_w(x, y)$ and $e_f(x, y), e_w(x, y)$ functions are generated, and the geometrical description at any (x, y) location inside the RUC is completed automatically for any member of the family $n_g/1/1$.

From Fig. 1 it can be observed that, for the present case of $n_g/1/1$ twill class, the fill and warp undulation shapes are identical, but for a minus sign, i.e.,

$$z_w(y,x;i) = -z_f(x,y;j) \quad for \quad i = j \tag{14}$$

Therefore, there is no need to separately define the warp function (10), because it is generated by (9) and (14) together with a change of variables from a_f, g_f, h_f to a_w, g_w, h_w .

The same applies for the cross-sectional functions (11) - (12); there is no need to define the warp thickness function $e_w(x; i)$ because it is generated based on

$$e_w(x;i) = e_f(y;j) \quad for \quad i = j \tag{15}$$

along with the aforementioned change of variables.

3.2 Twill weave family with $n_s = 1$

This class of twills is exemplified in Figs. 3 b,c and 6 a,b,c. The family is codified as $n_g/1/n_i$. This family is characterized by its ability to generate twill weaves with wider diagonal ribs than the previously described $n_g/1/1$ class. The $n_g/1/1$ geometry can be retrieved from the $n_g/1/n_i$ model by imposing $n_i = 1$, and the plain weave results from setting $n_g = 2$, $n_s = n_i = 1$. In this way, there is no need, other than didactical, for the $n_g/1/1$ model in Sect. 3.1, since that is a particular case of the $n_g/1/n_i$ model presented in this section.

The function describing the undulation $z_f(x, y)$ of the fill is given by

$$\begin{aligned} &\text{if } (j-1)w_f + \frac{g_f}{2} \le y \le jw_f - \frac{g_f}{2} \\ &z_f(x,y;j) = \\ & \left\{ \begin{array}{l} \frac{-h_f}{-h_f} \sin\left[\frac{\pi x}{w_w} + (j-1)\pi\right] \\ &\text{if } (j-\frac{3}{2})w_w \le x \le (j-\frac{1}{2})w_w \\ \frac{-h_f}{2} \\ &\text{if } (j-\frac{1}{2})w_w < x < (j-\frac{3}{2}+n_i) w_w \\ & (-1)^{n_i}\frac{h_f}{2} \sin\left[\frac{\pi x}{w_w} + (j-1)\pi\right] \\ &\text{if } (j-\frac{3}{2}+n_i) w_w \le x \le (j-\frac{1}{2}+n_i) w_w \\ & \frac{h_f}{2} \\ &\text{if } (j-\frac{1}{2}-n_g+n_i) w_w < x < (j-\frac{3}{2}) w_w \\ &\text{or } (j-\frac{1}{2}+n_i) w_w < x < (j-\frac{3}{2}+n_g) w_w \\ & (-1)^{(n_g+1)}\frac{h_f}{2} \sin\left[\frac{\pi x}{w_w} + (j-1)\pi\right] \\ &\text{if } (j-\frac{3}{2}-n_g) w_w \le x \le (j-\frac{1}{2}-n_g) w_w \\ & \text{or } (j-\frac{3}{2}+n_g) w_w \le x \le (j-\frac{1}{2}-n_g) w_w \\ & \text{or } (j-\frac{3}{2}+n_g) w_w \le x \le (j-\frac{1}{2}-n_g) w_w \\ & (-1)^{(n_g+1)}(-1)^{(n_i+1)}\frac{h_f}{2} \sin\left[\frac{\pi x}{w_w} + (j-1)\pi\right] \\ & \text{if } (j-\frac{3}{2}-n_g+n_i) w_w \le x \le (j-\frac{1}{2}-n_g+n_i) w_w \\ & \left(-\frac{h_f}{2}\right) \\ & \text{if } (j-\frac{1}{2}-n_g) w_w < x < (j-\frac{3}{2}-n_g+n_i) w_w \\ & \frac{-h_f}{2} \\ & \text{if } (j-\frac{1}{2}-n_g) w_w < x < (j-\frac{3}{2}-n_g+n_i) w_w \\ & \text{if } [(j-1)w_f \le y < (j-1)w_f + \frac{g_f}{2} \quad \text{or } jw_f - \frac{g_f}{2} < y \le jw_f] \\ & \text{and } g_f \ne 0 \\ & z_f(x,y;j) = 0 \end{aligned}$$

Unlike $n_g/1/1$ in Section 3.1, for this more general and broad family, the warp and fill undulations are not identical. This can be seen in Fig. 3 b and c, where it can be seen that the warp i = 1 does not have a similar undulation to the fill j = 1, the warp i = 2 is not like the fill j = 2, and so on. This means that equation (14) relating the undulation of warp to that of the fill cannot be applied, and the unknown $z_w(x, y)$ can not be automatically generated based on the $z_f(x, y)$ in (16), as it was the case of the $n_g/1/1$ model.

However, it can be observed in Fig. 3 b that the undulation of the warp i = 1 corresponds to that one of the fill j = 5, the warp i = 2 corresponds to the fill j = 1, the warp i = 3 corresponds to the fill j = 2, the warp i = 4 corresponds to the fill j = 3, and the warp i = 5 corresponds to the fill j = 4. Similarly, the correspondence between the undulation of i = (1, 2, 3, 4, 5) warps and j = (4, 5, 1, 2, 3) fills can be observed in Fig. 3 c. Of course, the word 'corresponds' used here assumes the change in sign in (14), and the change of variables from a_f, g_f, h_f to a_w, g_w, h_w as explained in Section 3.1.

The advantage of identifying a correspondence between the warp and fill undulation is exploited by defining a correspondence vector CV, which in the case of 5/1/2 twill (Fig. 3 b) is CV = (5, 1, 2, 3, 4), and in the case of 5/1/3 twill (Fig. 3 c) is CV = (4, 5, 1, 2, 3).

The rule for identifying the correspondence vector CV of any twill/satin weaving pattern is the following: the value of the element CV(i) $(i = 1 \cdots n_g)$ is the value of j for the fill that overlaps in the same position i as the *i*-th warp. Correspondence vectors for a number of weaves are given in Table 1.

Having the correspondence vector defined, the warp undulation $z_w(x, y)$ can be generated automatically in terms of the fill undulation $z_f(x, y)$ (14) and the correspondence vector CV, which assures a mapping between the fill and warp undulations. In this way, there is no need of write $z_w(x, y)$ separately, as it can be

			CV(i)					
n_g	n_s	n_i	1	2	3	4	5	6
2	1	1	1	2]			
3	1	1	1	2	3			
3	1	2	3	1	2			
3	2	1	1	2	3			
4	1	1	1	2	3	4		
4	1	2	4	1	2	3		
4	1	3	3	4	1	2		
4	2	1	-	-	-	-		
4	3	1	1	2	3	4		
5	1	1	1	2	3	4	5	
5	1	2	5	1	2	3	4	
5	1	3	4	5	1	2	3	
5	1	4	3	4	5	1	2	
5	2	1	1	5	4	3	2	
5	3	1	1	5	4	3	2	
5	4	1	1	2	3	4	5	
6	1	1	1	2	3	4	5	6
6	1	2	6	1	2	3	4	5
6	1	3	5	6	1	2	3	4
6	1	4	4	5	6	1	2	3
6	1	5	3	4	5	6	1	2
6	2	1	-	-	-	-	-	-
6	3	1	-	-	-	-	-	-
6	4	1	-	-	-	-	-	-
6	5	1	1	2	3	4	5	6

Table 1: Correspondence vectors CV for weaving patterns with $n_g = 2 \cdots 6$

derived from $z_f(x, y)$, which is already available in (16), thus simplifying the formulation.

The cross-sectional thickness functions are the same as those presented in (11), (12), with the correspondence relation (15) being applicable. The i, j parameters are similarly eliminated using (13), and the geometrical model of the $n_g/1/n_i$ twill family is completely defined.

3.3 Satin weave family with $n_i = 1$

This class of satin weaves is illustrated in Fig. 3 d, e, and 6 d,e. It can be observed that this model has the ability to describe satin weaves that feature any shift n_s between interlacing regions of two adjacent tows, while keeping the interlacing to $n_i = 1$. Similarly to the previous case, the $n_g/1/1$ twill geometry becomes a particular case of the $n_g/n_s/1$ model, by setting $n_s = 1$, and the plain weave results from setting $n_g = 2$, $n_s = n_i = 1$.

The function describing the fill undulation $z_f(x, y)$ is given by

$$\begin{cases} \text{ if } (j-1)w_f + \frac{g_f}{2} \le y \le jw_f - \frac{g_f}{2} \\ z_f(x,y;j) = \\ \begin{cases} \text{ for } k \in 1 \dots \text{trunc} \left[\frac{n_s(j-1)+1}{n_g} \right] + 1 \\ \left\{ \begin{array}{l} (-1)^{(kn_g+1)} \frac{h_f}{2} \sin\left[\frac{\pi x}{w_w} + n_s(j-1)\pi \right] \\ \text{ if } \left[n_s(j-1) - \frac{1}{2} - kn_g \right] w_w \le x \le \left[n_s(j-1) + \frac{3}{2} - kn_g \right] w_w \\ \frac{h_f}{2} \\ \text{ if } (k-1) \ge 1 \\ \text{ and } \left[n_s(j-1) + \frac{3}{2} - kn_g \right] w_w < x < \left[n_s(j-1) - \frac{1}{2} - (k-1)n_g \right] w_w \\ \frac{-h_f}{2} \sin\left[\frac{\pi x}{w_w} + n_s(j-1)\pi \right] \\ \text{ if } \left[n_s(j-1) - \frac{1}{2} \right] w_w \le x \le \left[n_s(j-1) + \frac{3}{2} \right] w_w \\ (17) \\ (-1)^{(n_g+1)} \frac{h_f}{2} \sin\left[\frac{\pi x}{w_w} + n_s(j-1)\pi \right] \\ \text{ if } \left[n_s(j-1) - \frac{1}{2} + n_g \right] w_w \le x \le \left[n_s(j-1) + \frac{3}{2} + n_g \right] w_w \\ \frac{h_f}{2} \\ \text{ if } \left[n_s(j-1) - \frac{1}{2} + n_g \right] w_w \le x \le \left[n_s(j-1) - \frac{1}{2} \right] w_w \\ \text{ or } \left[n_s(j-1) + \frac{3}{2} - n_g \right] w_w < x < \left[n_s(j-1) - \frac{1}{2} + n_g \right] w_w \\ \text{ if } \left[(j-1)w_f \le y < (j-1)w_f + \frac{g_f}{2} \\ \text{ or } \left[n_s(j-1) + \frac{3}{2} \right] w_w < x < \left[n_s(j-1) - \frac{1}{2} + n_g \right] w_w \\ \text{ if } \left[(j-1)w_f \le y < (j-1)w_f + \frac{g_f}{2} \\ \text{ or } jw_f - \frac{g_f}{2} < y \le jw_f \right] \\ \text{ and } g_f \ne 0 \\ z_f(x,y;j) = 0 \end{array}$$

Similarly to the model in Section 3.2, the warp undulation $z_w(x, y)$ is generated automatically from the fill undulation (17) based on the correlation equation (14) and the correlation vector CV. The cross-sectional thickness functions are the same as those presented in (11), (12), with the correspondence relation (15) being applicable, and the i, j parameters are eliminated using (13). The geometrical model of the $n_g/n_s/1$ satin family is completely defined by these elements.

Note the empty entries in in Table 1 for 4/2/1, 6/2/1, 6/3/1, and 6/4/1 weaves. These are empty because any member of the family $n_g/n_s/1$ with n_g and n_s having at least one common divisor (different from 1) identifies an invalid weave, featuring non-interlaced tows.

Some examples of woven fabric structures generated by the formulation proposed in Sections 3.1 - 3.3 are presented in Fig. 6. Prediction of some important geometrical quantities, such as crimp and maximum undulation angle, are presented in Section 4.

4 Results

A parametric study is presented next with the objective of elucidating the influence of two types of parameters on the 3D geometrical features of the fabric:

- Parameters defining the individual tow geometry: $a_f, g_f, h_f, a_w, g_w, h_w$ and
- Parameters defining the weaving pattern of the fabric reinforcement: n_q, n_s, n_i .

In this section, for simplicity, the fill and warp geometrical parameters are taken to be equal to each other. Therefore, the subscripts f, w, are not needed, and the parameters are thus referred simply as tow width a, tow thickness h, and gap g.

A range of gap to tow width ratio g/a = (0.05-1) and tow thickness to tow width ratio h/a = (0.05-0.5) is considered for the parametric study, for a fixed width a = 2 mm. The influence of the parameters g/a and h/a is illustrated for a plain weave 2/1/1. The influence of weaving pattern is illustrated for a twill family described by $n_g/1/1$ with $n_g = (1 \cdots 11)$.

The subscript notations m and M are be used in the following for the *minimum* and *maximum* values of the gap g and tow thickness h, respectively; at the extremes of their variation intervals.

The geometrical parameters that characterize the undulation of the woven fabric are:

- The maximum value θ_{max} of the undulation angle (2), and
- the crimp value c (4),

both calculated from the model. The undulation θ_{max} is expected to affect the failure initiation inside RUC, i.e., secondary local failure modes of the fill/warp tows, such as transverse tensile or shear failure, are expected to be induced earlier at the location of higher undulation angles θ_f, θ_w . The crimp value c is expected to affect the overall thermo-elastic properties of the composite material, i.e., a higher crimp value results in a higher knock-down factor of the thermo-elastic constants.

In Fig. 7, it can be observed that both, the maximum undulation θ_{max} and crimp c follow the same trend as a function of the gap/width ratio g/a. The same is true in Fig. 8 as a function of the thickness/width ratio h/a. Note the wide range of the maximum undulation angle θ_{max} and crimp value c (from 37° to 2°, and from 12% to almost 0%, respectively).

Further, it can be noticed in Fig. 9 that the weaving style variable n_g has no influence on θ_{max} , but it has a strong influence on the crimp value, especially for the case of tows with high cross-sectional thickness h and small gap g, and especially in the first half of the n_g range, which is more common in practical applications.

5 Conclusions

First, the geometrical model proposed in Sections 3.2–3.3 is able to cover a large variety of cases. However, not all the possible weaving styles are represented. For example, a 7/2/3 twill pattern is not described by the twill model in Section 3.2, which however is able to describe all the cases in the family $n_g/1/n_i$. Furthermore, a satin pattern 7/3/2 is not described by the satin model in Section 3.3, which however is able to describe all the members of the family $n_g/n_s/1$. However, none of the models in the literature is able to claim this level of generality, but only very particular cases of twill and satin are presented elsewhere, without achieving the level of generality of the present model.

Second, the present geometrical model is formulated at ply level, which means that one single reinforcing layer is generated and consequently analyzed by the model, as being representative for the whole laminate. This fact has to be regarded as an approximation of the model, based on the assumption that the stacking sequence of a laminate would feature an *in-phase* configuration, as depicted in Fig. 10; only in this case a single ply can be selected and regarded as a repetitive unit through the thickness of the laminate. For the case of *out-of-phase* stacking sequence, a through the thickness RUC including multiple plies has to be considered for analysis, as it is done in [11, 12] for the case of plain weave only. The assumption of in-phase laminate stacking sequence also implies that the nesting effect in between adjacent plies due to the compaction during consolidation of the composite material is neglected, which might have an effect on the evaluation of the fiber volume fraction inside RUC and fiber volume fraction inside individual tows, both of which are primary parameters for the mechanical analysis of the composite material. In order to consider the ply nesting effect due to compaction, the in-phase assumption has to be dropped, and a RUC selection similar to [3, 4, 6, 8-10] has to be considered, noting that [3, 4, 6, 8-10] dealt with plain weave only.

It can be noted that the out-of-phase and nesting effects are difficult to implement in the present model, whose goal is to provide a high level of generality regarding the weaving patterns covered. However, if the present geometrical model is used as a CAD tool in a numerical (FEA) model of mechanical properties, then ply nesting and out-of-phase can be more or less easily implemented by shifting individual plies with the CAD tool. Even more involved analysis could be performed in a FEA context, not only considering the out-of-phase and nesting between different plies of the laminate, but also having the possibility to consider laminates where individual plies are stacked on different faces (the top or bottom face of a ply), or even the Third, smaller representative volume elements (RVE) than the RUC can be selected in some cases at the expense of limiting the applicability of the models to specific weaving patterns for which the selected RVE applies [4, 17, 27, 33]. The whole RUC method is considered in the present work in order to preserve the ability to model all the weave patterns with a single analytical formulation.

Finally, a closer analysis of the sinusoidal undulation and cross-sectional functions presented in Section 3.1 – 3.3 reveals the fact that matrix wedges layers are simulated in between crossing fill and warp tows (labeled I and II in Fig. 5). These matrix layers has been addressed in [8], where the criteria for a correct geometrical model are defined as: (a) continuity of tows in both fill and warp directions (which means continuous $z_f(x, y), z_w(x, y)$ functions), (b) continuity of tow slopes in both fill and warp directions (which means continuous $\theta_f(x, y), \theta_w(x, y)$ functions), and (c) perfect contact between fill and warp tows at the cross-overs of the undulated regions (region I in Fig. 5). It can be noted that the last condition is not satisfied by the present model. However, the matrix layers labeled II in Fig. 5 (at cross-overs of non-undulated regions) are in conformity with the photomicrographical observation, and are considered a model refinement in [24]. Still, the matrix layers labeled I, however small they are, are artificially induced and may be regarded as a model drawback. The thickness of the matrix wedge I is small and it is not expected to have considerable effect on the mechanical properties, but one might still want to correct such imperfection if the present geometrical model is used for mesh generation of FEA analysis. Such minor imperfections could be corrected in a similar manner to [8]. However, the model in [8] applies to plain weave only, and it does not meet condition (b) in the warp direction, as stated by the same reference [8].

In summary, the proposed formulation provides analytical expressions to completely define the internal 3D geometry of the majority of practical cases of 2D fabrics including plain weave, twill, and satin. The complete set of geometrical properties for the reinforcing architecture (e.g., tow length, cross-section area, volume, local undulation angle, and crimp value) are calculated in order to further characterize the mechanical behavior of the fabric reinforced composite material. Applications of the proposed geometrical model for stiffness and strength prediction will be addressed in *Part II* of this manuscript.



Figure 6: Generated twill and satin weaves: (a) 7/1/1 twill weave; (b) 7/1/2 twill weave; (c) 7/1/3 twill weave; (d) 7/2/1 satin weave; (e) 7/3/1 satin weave.



Figure 7: Variation of θ_{max} and c undulation parameters with gap g.



Figure 8: Variation of θ_{max} and c undulation parameters with tow thickness h.



Figure 9: Variation of θ_{max} and c undulation parameters with weaving style.



Figure 10: In-phase through the thickness configuration of the composite laminate.

References

- [1] B. Cox, Handbook of analytical methods for textile composites, Tech. Rep. NASA Contractor Report 4750, NASA Langley Research Center (March 1997).
- [2] E. J. Barbero, Introduction to Composite Materials Design Second Edition, CRC, Boca Raton, FL, 2010.
- [3] N. Naik, V. Ganesh, Prediction of on-axis elastic properties of plain weave fabric composites, Compos Sci Technol 45 (1992) 135–152.
- [4] N. Naik, Woven Fabric Composites, Technomic, Lancaster, PA, 1994.
- [5] N. K. Naik, V. K. Ganesh, Failure behavior of plain weave fabric laminates under in-plane shear loading, Journal of Composites Technology and Research 16 (1) (1994) 3–20.
- [6] N. K. Naik, V. K. Ganesh, Analytical method for plain weave fabric composites, Composites 26 (4) (1995) 281–289.
- [7] V. K. Ganesh, N. K. Naik, Failure behaviour of plain weave fabric laminates under in-plane shear loading: Effect of fabric geometry, Composite Structures 30 (2) (1995) 179–192.
- [8] N. Naik, V. Ganesh, Failure behavior of plain weave fabric laminates under on-axis uniaxial tensile loading: I - Laminate geometry, J Compos Mater 30 (16) (1996) 1748–1778.
- [9] N. Naik, V. Ganesh, Failure behavior of plain weave fabric laminates under on-axis uniaxial tensile loading: II - Analytical predictions, J Compos Mater 30 (16) (1996) 1779–1882.
- [10] N. Naik, V. Ganesh, Failure behavior of plain weave fabric laminates under on-axis uniaxial tensile loading: III - Effect of fabric geometry, J Compos Mater 30 (16) (1996) 1883–1856.
- [11] M. Ito, T.-W. Chou, Elastic moduli and stress field of plain weave composites under tensile loading, Compos Sci Technol 57 (1997) 787–800.
- [12] M. Ito, T.-W. Chou, An analytical and experimental study of strength and failure behavior of plain weave composites, J Compos Mater 32 (1) (1998) 2–30.
- [13] J. Zeman, M. Sejnoha, Homogenization of balanced plain weave composites with imperfect microstructure: Part I : Theoretical formulation, International Journal of Solids and Structures 41 (22-23) (2004) 6549–6571.
- [14] A. Adumitroaie, E. J. Barbero, Stiffness and strength prediction for plain weave textile reinforced composites, accepted, Mechanics of Advanced Materials and Structures xx (xx) (2010) xx.
- [15] T. Ishikawa, T. W. Chou, Stiffness and strength behavior of woven fabric composites 71 (1982) 3211– 3220.
- [16] T. Ishikawa, T.-W. Chou, Elastic behavior of woven hybrid composites, Journal of Composite Materials 16 (1982) 2–19.
- [17] I. S. Raju, J. T. Wang, Classical laminate theory models for woven fabric composites, Journal of Composites Technology and Research 16 (4) (1994) 289–303.
- [18] R. A. Naik, Analysis of woven and braided fabric reinforced composites, Tech. Rep. NASA Contractor Report 194930 (June 1994).
- [19] R. A. Naik, Failure analysis of woven and braided fabric reinforced composites, Journal of Composite Materials 29 (17) (1995) 2334–63.
- [20] D. Scida, Z. Aboura, Prediction of the elastic behavior of hybrid and non-hybrid woven composites, Compos Sci Technol 59 (1997) 1727–1740.

- [21] D. Scida, Z. Aboura, A micromechanics model for 3D elasticity and failure of woven fiber composite materials, Compos Sci Technol 59 (1999) 505–517.
- [22] S. Z. Sheng, S. V. Hoa, Three dimensional micro-mechanical modeling of woven fabric composites, J Compos Mater 35 (2001) 1701–1729.
- [23] Z.-M. Huang, S. Ramakrishna, Towards automatic designing of 2D biaxial woven and braided fabric reinforced composites, Journal of Composite Materials 36 (13) (2002) 1541–79.
- [24] M. P. Rao, M. Pantiuk, P. G. Charalambides, Modeling the geometry of satin weave fabric composites, Journal of Composite Materials 43 (1) (2009) 19–56.
- [25] P. Vandeurzen, J. Ivens, I. Verpoest, A three-dimensional micromechanical analysis of woven-fabric composites. I. Geometric analysis, Composites Science and Technology 56 (11) (1996) 1303–15.
- [26] J. L. Kuhn, P. G. Charalambides, Modeling of plain weave fabric composite geometry, Journal of Composite Materials 33 (3) (1999) 188–220.
- [27] T.-W. Chou, Microstructural Design of Fiber Composites, Cambridge University Press, Cambridge, MA., 1992.
- [28] Z.-M. Huang, Efficient approach to the structure-property relationship of woven and braided fabricreinforced composites up to failure, Journal of Reinforced Plastics and Composites 24 (12) (2005) 1289–1309.
- [29] J. A. Hewitt, D. Brown, R. B. Clarke, Computer modelling of woven composite materials, Composites 26 (2) (1995) 134–140.
- [30] P. Vandeurzen, J. Ivens, I. Verpoest, A three-dimensional micromechanical analysis of woven-fabric composites. II. Elastic analysis, Composites Science and Technology 56 (11) (1996) 1317–27.
- [31] E. J. Barbero, T. M. Damiani, J. Trovillion, Micromechanics of fabric reinforced composites with periodic microstructure, Int J Solids Struct 42 (2005) 2489–2504.
- [32] E. J. Barbero, J. Trovillion, J. Mayugo, K. Sikkil, Finite element modeling of plain weave fabrics from photomicrograph measurements, Composite Struct 73 (1) (2006) 41–52.
- [33] X. Tang, J. D. Whitcomb, Progressive failure behaviors of 2d woven composites, Journal of Composite Materials 37 (14) (2003) 1239–1259.