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# A micro-mechanics damage approach for fatigue of composite materials

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## Abstract

A new model for fatigue damage evolution of polymer matrix composites (PMC) is presented. The model is based on a combination of an orthotropic damage model and an isotropic fatigue evolution model. The orthotropic damage model is used to predict the orthotropic damage evolution within a single cycle. The isotropic fatigue model is used to predict the magnitude of fatigue damage accumulated as a function of the number of cycles. This approach facilitates the determination of model parameters since the orthotropic damage model parameters can be determined from available data from quasi-static-loading tests. Then, limited amount of fatigue data is needed to adjust the fatigue evolution model. The combination of these two models provides a compromise between efficiency and accuracy. Decomposition of the state variables down to the constituent scale is accomplished by micro-mechanics. Phenomenological damage evolution models are then postulated for each constituent and for the micro-structural interaction among them. Model parameters are determined from available experimental data. Comparison between model predictions and additional experimental data is presented. © 2002 Published by Elsevier Science Ltd.

Keywords: Damage; Fatigue; Composite materials; Micro-mechanics; Periodic microstructure; Strain concentration; Stress concentration

# 1. Introduction

Various fatigue damage models can be classified according to the complexity of the material they model (isotropic, transversely isotropic, orthotropic) and by the order of the tensor representing the damage measure (scalar, vector, second-order tensor, and so on). Long fiber reinforced polymer matrix composites (PMC) are strongly orthotropic and they damage in an orthotropic fashion as well [18,19,23]. Models capable of representing such a general situation require many experimental parameters and high computational effort.

Fatigue damage of metal matrix composites has been modeled using a scalar damage measure and considering the material to be homogeneous and transversely isotropic [3]. Such models are efficient in that a limited number of tests are sufficient to determine the model parameters, but all tests need to be fatigue tests, which may be difficult and time consuming. On the other hand, quasi-static-loading damage models can be generalized for fatigue loading by introducing cyclic hardening [32–

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35]. That is, the hardening law is generalized to include a dependency on the number of cycles N. Such models usually require detailed computation of each and every cycle during the life of the material, which may be computationally expensive. A compromise between these two approaches is sought in this paper.

Using classical laminate theory and other similar approaches, PMC are routinely analyzed by assembling the laminae stiffness response into laminate stiffness response [6]. The laminate response to external loads is then decomposed into laminae responses; that is, the point stress and strains on the homogeneous orthotropic lamina material are found. Similarly, micro-mechanics allows us to predict lamina stiffness from constituent (fiber, matrix, and interphase) properties [6]. Some micromechanical models [2,11,20,32-35] make it possible to decompose the lamina state variables (e.g., stress, strain, damage) into their components in each of the constituents. Therefore, damage evolution models and failure criteria can be formulated at the constituent level. The shortcoming of this approach is that only distributed damage with a length scale much shorter than the characteristic dimensions of the micro-structure should be modeled by continuous damage mechanics.

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Otherwise, interaction effects between the constituents due to relatively large cracks in the individual phases are neglected at the constituent level. Such interaction effects are recovered herein by adding an extra step at the lamina level. The resulting integrated model accounts for different initiation, evolution, and failure of the two main phases, fiber and matrix. The additional lamina model accounts for any effects not captured by the constituent models.

Regarding the assumption of distributed damage, the following comments are in order. The characteristic length of a material element over which the stress and strains do not change rapidly is the lamina thickness. The fiber diameter, fiber spacing, and dimensions of micro-cracks are much smaller than the lamina thickness. Therefore, fiber breaks, matrix crazes and microcracks can be analyzed as distributed damage. Since, damage of the fiber phase contributes only to loss of stiffness and strength in the fiber direction, the characteristic length of the fiber phase is of the order of the fiber length, supporting the assumption that fiber breaks can be modeled as distributed damage [15,28]. Transverse matrix cracks spanning the lamina thickness cannot be considered as distributed damage [12,24,26,27, 29]. Therefore, a lamina-level model was added to account for those effects, always in an average sense. An alternative option would be to formulate the quasi-static model completely at the lamina level [5,7,9,17]. Such approach requires a more extensive database of lamina strength values. In this work, it was decided to use the constituent level approach because of the basic nature of the strength data required, namely fiber strength, Weibull dispersion, and so on.

Due to the orthotropic nature of the damage measure  $D_{ij}$  used in this work, loss of transverse isotropy of a lamina due to damage can be predicted. The material parameters controlling damage evolution in each phase and their interaction, within a loading cycle, are determined from quasi-static-loading tests, which are available [10,17,31]. Then, a S-N curve of a single laminate is used to adjust two additional parameters governing the

Table 1				
Material	properties	for	T300/5208	

Property	Fiber	Matrix	Lamina
Modulus, E (GPa)	230	2.6	
Poisson's ratio v	0.22	0.38	0.268
Initial volume fraction	0.7	0.3	-
$F_{\rm t}$ (GPa)	3.654	0.0586	0.0586 (trans-
F <sub>c</sub> (GPa)	1.096	0.1876	verse) 1.096 (longi-
Critical D <sub>t</sub>	0.105161	0.5	(udinal)
Critical D <sub>c</sub>	0.110945	0.5	0.110945
F <sub>6</sub> (GPa)	-	_	0.08616
G <sub>12</sub> (GPa)	104.545	0.971	5.090
Weibull dispersion m	8.9	_	-

evolution of the magnitude of fatigue damage as a function of the number of cycles. Finally, predicted S-N curves for other laminates are compared with their corresponding test data. The only problem encountered was to find fatigue data for the same materials systems for which comprehensive quasi-static data exists (the data required is listed in Table 1).

# 2. Damage

# 2.1. Quasi-static damage model

The first building block of the proposed approach is a damage model for a single loading cycle. The model is based on the combination of two continuous damage models at the constituent level and one continuous damage model at the lamina level. For each phase, damage is represented by a state variable, in the form of a second-order damage tensor  $D_{ij}$ , or by its complement the integrity tensor  $\Omega_{ij}$  [16]. Since tension, compression, and shear have different effects on damage, crack closure coefficients  $c_n$  and  $c_s$ , for compression and shear respectively, are introduced in the definition of effective stress, as follows:

$$\overline{\sigma}_{ij} = M_{ijkl}\sigma_{kl},\tag{1}$$

where

$$M_{ijkl} = \frac{\zeta_{ijkl}\langle\sigma_{kl}\rangle}{1 - D_{ij}} + \frac{\zeta_{ijkl}\langle-\sigma_{kl}\rangle}{1 - c_{n}D_{ij}} + \frac{I_{ijkl} - \zeta_{ijkl}}{2} \left[\frac{1}{1 - c_{n}D_{ii}} + \frac{1}{1 - c_{s}D_{ii}}\right]$$
(2)

and  $\zeta_{ijkl} = 1$  if i = j = k = l = 1, 0 otherwise, and the brackets  $\langle \rangle$  represent the Heaviside function. Such definition assures symmetry of the effective stress tensor. The effect of crack closure on the normal and shear components of effective stress can be adjusted from no effect to full effect by varying the coefficients  $c_n$ ,  $c_s$ , from 0 to 1.

The stiffness tensor C of one configuration (say damaged) is obtained in terms of the stiffness  $\overline{C}$  of the precursor configuration (say virgin) by using the energy equivalence principle [16], as

$$C = M^{-1} : \bar{C} : M^{-1}.$$
(3)

The analysis involves three configurations [32-35]: effective  $(\bar{\sigma})$ , partially damaged  $(\bar{\sigma})$  and damaged  $(\sigma)$ . In the effective configuration, the undamaged portion of fiber and matrix carry the load. In the partially damaged configuration, the fiber and matrix are damaged but the interaction damage is not present. In the damaged configuration, all the damage is present. The three configurations are illustrated in Fig. 1, starting with



Fig. 1. Damage configurations, from left to right, effective, partially damaged, and damaged.

damaged on the right, partially damaged at the center, and effective on the left.

Mapping between configurations is accomplished by the appropriate damage effect tensor,  $M^L$  from damaged to partially damaged (due to interaction damage effects),  $M^f$  and  $M^m$  from partially damaged to effective for fiber and matrix phases, respectively. The total damage-effect tensor M that accounts for the combined effect of fiber, matrix, and interaction damage is given by

$$M_{ijrs} = (c^{\mathrm{f}} M_{ijkl}^{\mathrm{f}} B_{kluv}^{\mathrm{f}} + c^{\mathrm{m}} M_{ijkl}^{\mathrm{m}} B_{kluv}^{\mathrm{m}}) M_{uvrs}^{\mathrm{L}}.$$
 (4)

At each configuration, mapping between phases (fiber, matrix, and lamina) is accomplished by micromechanics using the stress and strain concentration tensors, B and A respectively, according to

$$\sigma^{\mathrm{r}} = B^{\mathrm{r}} : \sigma, \quad B^{\mathrm{r}} = C^{\mathrm{r}} : A^{\mathrm{r}} : C^{-1},$$
  

$$\varepsilon^{\mathrm{r}} = A^{\mathrm{r}} : \varepsilon, \quad A^{\mathrm{r}} = C^{-\mathrm{f}} : B^{\mathrm{r}} : C,$$
(5)

where  $\sigma^r$  indicates the average stress in the phase r = f, m, L, and  $\sigma$  is the average stress in the homogenized material. Eq. (5) account for the stress redistribution between the fiber and the matrix that must take place when both phases undergo damage at different rates. Stress redistribution also takes place at the macro-level (among laminae) as a result of updating of the lamina stiffness tensor C according to Eq. (3).

The stress and strain concentration tensors are obtained using [20] in the effective configuration. Then, the concentration tensors in the partially damaged configuration are computed as:

$$\widetilde{A}^{\tilde{f}} = [v^{m}M^{-m}\overline{A}^{m}\overline{A}^{-f}M^{-f} + v^{f}I]^{-1},$$

$$\widetilde{A}^{m} = \frac{1}{v^{m}}[I - v^{f}\widetilde{A}^{f}],$$

$$I_{ijkl} = \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}).$$
(6)

Similar equations are used to compute the concentration tensors in the damaged configuration in terms of the same in the partially damaged one. Also for each configuration, the stiffness tensor of the homogenized material is computed by micro-mechanics as

$$C = v^{\mathrm{f}} C^{\mathrm{f}} : A^{\mathrm{f}} + v^{\mathrm{m}} C^{\mathrm{m}} : A^{\mathrm{m}}, \tag{7}$$

where  $v^{f}$  and  $v^{m}$  are the fiber and matrix volume fractions of the current configuration. The volume fractions in the damaged configuration are those determined during fabrication of the composite. The volume fractions in the effective configuration are different than those in the damaged configuration because the effective configuration deals with the volume of undamaged fiber and matrix, both of which are different from the original volumes of fiber and matrix of the as-produced composite. The fiber and matrix volume fractions in the effective configuration are computed by taking into account the amount of damaged volume in the fiber and matrix phases [8], as follows:

$$\overline{v}^{\rm r} = \frac{v^{\rm r}(1 - D_{\rm eq}^{\rm r})}{v^{\rm f}(1 - D_{\rm eq}^{\rm f}) + v^{\rm m}(1 - D_{\rm eq}^{\rm m})},$$

$$D_{\rm eq}^{\rm r} = (D_{ij}^{\rm r} D_{ij}^{\rm r})^{1/2}.$$
(8)

Parameter	Fiber	Matrix	Lamina
$ \begin{array}{l} H_{1} \\ H_{2} = H_{3} \\ J_{11} \\ J_{22} = J_{33} \end{array} $	$\begin{array}{c} 0.26300364 \times 10^{-13} \\ 0.26300364 \times 10^{-13} \\ -0.14275391 \times 10^{-5} \\ -0.14275391 \times 10^{-5} \end{array}$	$\begin{array}{c} 0.214158 \times 10^{-13} \\ 0.214158 \times 10^{-13} \\ -0.61699 \times 10^{-6} \\ -0.61699 \times 10^{-6} \end{array}$	$\begin{array}{c} 0.14713 \times 10^{-10} \\ 0.7319 \times 10^{-13} \\ -0.00003137 \\ 0.1296912 \times 10^{-8} \end{array}$

 Table 2

 Intermediate coefficients in terms of properties in Table 1

#### 2.2. Damage evolution

A damage surface is assumed to limit the space of thermodynamic forces Y for which no damage occurs. In this work, an off-centered surface is used [5] to account for different behavior in tension and compression of each phase, as follows:

$$g(Y,\gamma) = \sqrt{Y:J:Y} + \sqrt{|H:Y|} - (\gamma + \gamma_0), \qquad (9)$$

where J and H are second-order tensors of material coefficients which are univocally related to the material properties of each phase,  $\gamma$  is the damage parameter in Y-space, and  $\gamma_0$  is the damage threshold in Y-space. The second-order tensor Y contains the thermodynamic forces, dual to the damage tensor, in the thermodynamic sense as

$$Y = \frac{\partial \psi}{\partial D} = \frac{1}{2} \frac{\partial}{\partial D} [C : \varepsilon : \varepsilon], \qquad (10)$$

where the free energy  $\psi$  is given by the sum of the strain energy  $\pi$  plus the damage dissipation potential  $\Gamma$ , as

$$\psi = \pi(\varepsilon, D) + \Gamma(\delta), \tag{11}$$

where  $\delta$  is the damage parameter. Based on experimental observations [25,36], it is possible to assume that the damage principal directions coincide with the principal material directions, in which case the damage and integrity tensors become diagonal. With this simplification, it is possible to derive explicit equations relating the thermodynamic forces to the stress components. For example, using contracted notation [6] and a state of plane stress we have

$$Y_{1} = \frac{1}{\Omega_{1}^{2}} \left( \frac{\overline{C}_{11}}{\Omega_{1}^{4}} \sigma_{1}^{2} + \frac{\overline{C}_{12}}{\Omega_{1}^{2} \Omega_{2}^{2}} \sigma_{1} \sigma_{2} + \frac{\overline{C}_{66}}{\Omega_{1}^{2} \Omega_{2}^{2}} \sigma_{6}^{2} \right),$$

$$Y_{2} = \frac{1}{\Omega_{2}^{2}} \left( \frac{\overline{C}_{22}}{\Omega_{2}^{4}} \sigma_{2}^{2} + \frac{\overline{C}_{12}}{\Omega_{1}^{2} \Omega_{2}^{2}} \sigma_{1} \sigma_{2} + \frac{\overline{C}_{66}}{\Omega_{1}^{2} \Omega_{2}^{2}} \sigma_{6}^{2} \right),$$

$$Y_{3} = 0.$$
(12)

Using these explicit equations, the damage surface can be written in stress space, and its shape is the same as that of the Tsai-Wu quadratic failure criterion, but its size is variable as controlled by the magnitudes of the damage threshold and damage parameter. In summary, Eq. (9) reduces to

$$g = f_{ij}\sigma_i\sigma_j + f_i\sigma_i - (\gamma + \gamma_0) \quad (i = 1, 2, \dots, 6).$$
(13)

Eq. (13) coincides with the Tsai-Wu criterion when  $\gamma + \gamma_0 = 1$ . Since the Tsai-Wu criterion predicts lamina failure in terms of available strength data, it is possible to determine all the coefficients of the tensors J and H in Eq. (9) in terms of available strength data for each phase (see Table 1), as described in [1,5]. The numerical values are given in Table 2.

The size of the damage surface in Y-space evolves according to the following equation:

$$\gamma = \frac{\partial \psi}{\partial \delta} = c_1 [1 + \exp(-\delta/c_2)] \tag{14}$$

in terms of two empirical parameters  $c_1$  and  $c_2$  to be determined from experiments, with the damage parameter changing according to

$$\mathrm{d}\delta = \mu \frac{\mathrm{d}g}{\mathrm{d}\gamma} = -\mu. \tag{15}$$

The damage multiplier is found interactively so that the consistency conditions g = 0 and dg = 0 are met. That is, after an increment of strain that causes damage, the Y-state must remain on the g = 0 surface with no further change of that surface (dg = 0). Once the damage surface is reached, damage accumulates along the normal to the damage-flow surface defined as

$$f(Y,\gamma) = \sqrt{Y:J:Y} - (\gamma + \gamma_0).$$
(16)

The magnitude of additional damage is controlled by the damage multiplier so that

$$\mathrm{d}D_{ij} = \mu \frac{\partial f}{\partial Y_{ij}}, \quad D_{ij} = \int \mathrm{d}D_{ij}, \quad \Omega_{ij} = \delta_{ij} - D_{ij}. \tag{17}$$

A transversely isotopic lamina may become orthotropic as a result of damage, which is allowed in the formulation by virtue of Eq. (3) using an orthotropic damage tensor D and damage-effect tensor M. Also, the model separates damage in the fiber and matrix, and redistributes the stress into the fiber and matrix.

The quasi-static-loading model described in this section is capable of predicting damage accumulation for one cycle only. Since the evolution of the damage surface, given by Eq. (14), is independent of the number of cycles, no further damage can occur during cyclic loading at constant strain amplitude. A phenomenological model is therefore necessary to predict the accumulation of damage during cyclic loading, as it is described in Section 3.

### 3. Cyclic loading

Use of a scalar fatigue damage evolution reduces the amount of fatigue data needed to a minimum, since only two parameters need to be adjusted in the isotropic damage model. Stress redistribution that results from orthotropic damage is recovered by re-computing the relative magnitude of damage among the three principal material directions. Stress redistribution among the phases is also accomplished by distributing the scalar damage into the contributions of fiber, matrix, and interaction effects.

The scalar model of Arnold and Kruch [3] was used to track the magnitude of damage accumulated as a function of the number of cycles n. The number of cycles to failure N is predicted as

$$N(n) = \frac{1 - \left[1 - \Omega_n^{\beta - 1}\right]^{1 - \alpha}}{F_m^{\beta}(1 - \alpha)(\beta - 1)},$$
(18)

where  $\alpha$  and  $\beta$  are parameters adjusted using an experimental S-N curve of a laminate,  $\Omega_n$  is the scalar magnitude of the integrity tensor, computed from Eq. (17) as

$$\Omega_n = (\Omega_{ij}\Omega_{ij})^{1/2} \tag{19}$$

and the normalized stress amplitude  $F_m$  is defined in [3] in terms of experimental parameters defined in Table 4.

From Eq. (18), the scalar magnitude of the damage tensor can be calculated as

$$D_n = 1 - \left[1 - \left(\frac{n}{N}\right)^{1/(1-\alpha)}\right]^{1/(1+\beta)}.$$
 (20)

The magnitude of fatigue damage  $D_n$  is used to update the magnitude of the orthotropic damage tensor as

$$D_{ij}(n+\delta n) = \frac{D_n(n+\delta n)}{D_n(n)} D_{ij}(n).$$
<sup>(21)</sup>

The next step is to decompose the damage into fiber and matrix damage. Experimental data suggest that the rate of damage accumulation dD/dn is approximately constant for most of the fatigue life, with faster accumulation near fracture [12]. Therefore, it is possible to assume that the total increment of damage dD decomposes into fiber and matrix damage (r = f, m) as

$$\mathrm{d}D_{kl}^{\mathrm{r}} = \alpha_{ijkl}^{-\mathrm{r}} \,\mathrm{d}D_{ij},\tag{22}$$

where  $\alpha^r$  is the damage decomposition tensor for phase r = f, m, L, which is derived as follows. Start with the consistency condition

$$\mathrm{d}D_{ij} = \mu = \frac{(\partial g/\partial Y_{mn}) \,\mathrm{d}Y_{mn}}{(\partial g/\partial \gamma)^2 (\partial \gamma/\partial \delta)} \frac{\partial f}{\partial Y_{ij}}.$$
 (23)

Then, substitute the thermodynamic forces as

$$\mathrm{d}Y_{mn} = \frac{\partial Y_{mn}}{\partial \sigma_{pq}} \mathrm{d}\sigma_{pq}. \tag{24}$$

Since Eq. (24) can be evaluated explicitly from Eq. (12), it is possible to write Eq. (23) as

$$\mathrm{d}D_{ii} = X_{ijpq} \,\mathrm{d}\sigma_{pq} \tag{25}$$

in terms of the damage-stress tensor X. Using Eq. (25) and the average stress theorem at the lamina level

$$dD^{L} = X^{L} : d\sigma = X^{L} : (v^{f} d\sigma^{f} + v^{m} d\sigma^{m}).$$
<sup>(26)</sup>

Substituting the fiber and matrix stress we get

$$dD^{L} = X^{L} : (v^{f}X^{-f} : dD^{f} + v^{m}X^{-m} : dD^{m}).$$
(27)

Using the chain rule

$$\frac{\partial D^{L}}{\partial D^{r}} = v^{r} X^{L} : X^{-r} \quad (r = f, m).$$
<sup>(28)</sup>

Then, using Eq. (28) into (27) we get the damage decomposition tensor for phase r = f, m as

$$\alpha^{\rm r} = \frac{\partial D}{\partial D^{\rm r}} = \frac{\partial D}{\partial D^{\rm L}} : \frac{\partial D^{\rm L}}{\partial D^{\rm r}} = \text{constant}, \tag{29}$$

which can be used to decompose the total damage into the damage in each phase according to Eq. (22) [1]. A procedure to evaluate the derivatives in Eq. (29) is given in Appendix A.

#### 4. Computational procedure

The computational procedure is illustrated in Fig. 2. First, for a fixed strain amplitude S, one loading cycle is analyzed using the quasi-static model described in Section 2 to determine the orthotropic damage tensor  $D^{r}$  of each phase, the damage decomposition tensor  $\alpha^{r}$  of each phase (Eq. (29)), and the estimated life N of the lamina (Eq. (18)).

A finite increment of the number of cycles  $\delta n$  is then selected. This can be selected as the lowest remaining life in the structure, as the number of cycles to the next stress amplitude change, or any other user selected number of cycles. The magnitude of the damage tensor is then computed with Eq. (20). The orthotropic damage



Fig. 2. Computational procedure.



Fig. 3. Predicted and experimental, quasi-static, shear stress-strain curve for carbon/epoxy T300/5208 unidirectional lamina.

Table 3

tensor of the lamina is then scaled up with Eq. (21). Then, the damage tensor is decomposed into the phases using Eq. (22).

At this point, with n cycles applied, the quasi-static model is used again to refine the damage tensor  $D^r$  of each phase. The stress is redistributed using Eq. (3) at the laminate level and using Eq. (5) at the constituent level.

The procedure is repeated until the remaining life is equal to zero. The pair (S, N) obtained in this way is used to plot one point in the predicted S-N curve. The procedure is repeated for several values of strain amplitude S until the complete S-N curve is generated. The predicted curve is then compared with the experimental S-N curve and the parameters  $\alpha$  and  $\beta$  in Eq. (18) are adjusted to minimize the difference.

Nine model parameters (three per phase) are necessary to track the evolution of damage within one loading

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Parameter	Fiber	Matrix	Lamina	
<b>C</b> 1	1.0	1.0	1.0	
- 1 C7	$-1.1 \times 10^{5}$	$-4.2 \times 10^{6}$	-1.5	
-2 Va	-6.5	2.0	0.0	
70 Cn	1.0	1.0	1.0	
C ,	1.0	1.0	1.0	

cycle. For each phase (fiber, matrix, and lamina), there are three parameters:  $c_1^r$ ,  $c_2^r$ , in the evolution law Eq. (14) and the damage threshold  $\gamma_0^r$  in Eq. (9). The nine parameters are determined by modeling quasi-static-loading tests for which data are available. The procedure is illustrated using available data, shown in Table 1 for T300/5208 carbon/epoxy composite [10].

First, a longitudinal tensile test (ASTM D3039 [4]) is simulated. The fiber parameters  $(c_1^f, c_2^f, \gamma_0^f)$  are adjusted

Table 4			
Fatigue	parameters	for	T300/5208

Parameter	Symbol	Value	Method of determination
Static tensile strength (GPa)	σ	1.55	Experimental
Longitudinal to transverse normal strength ratio	ω,	29	Experimental
Longitudinal to transverse shear strength ratio	n.,	2.5	Experimental
Longitudinal to transverse normal fatigue limit ratio	Ŵn	12.48	Experimental
Longitudinal to transverse normal normalizing stress amplitude ratio	ω <sub>m</sub>	12.0	Experimental
Longitudinal to transverse shear fatigue limit ratio	no	0.8	Experimental
Longitudinal to transverse shear normalizing stress amplitude ratio	n	0.95	Experimental
Edigitudinal to transverse shour normanizing stress amplitude that	σ <sub>n</sub>	148	Experimental
Normalizing stress amplitude (GPa)	M <sub>m</sub>	2.5	Experimental
Fatigue life coefficient	α	0.2	Adjusted with laminate $S-N$
I aligue no coencient	-		curve
Fotigue life exponent	ß	12.6	Adjusted with laminate $S-N$
Taligue me exponent	r		curve



Fig. 4. Predicted and experimental S-N curve for [45/-45]s carbon/epoxy T300/5208.

so that at failure the fiber stress equals the fiber strength  $F_t^f$  and the fiber damage equals the known value  $D_t^f = 1 - \exp(-1/m)$ , where *m* is the Weibull modulus [15,28], which is available [14,21,22,30] (Table 1).

failure the transverse stress equals the known transverse strength of the composite  $F_t^m$  and the matrix damage equals  $D_t^m = 1/2$  as estimated by [13] (Table 1).

Second, a transverse tensile test is simulated [4]. The matrix parameters  $(c_1^m, c_2^m, \gamma_0^m)$  are adjusted so that at

Next, adjust the lamina parameters  $(c_1^L, c_2^L, \gamma_0^L)$  to minimize any discrepancies in the shape of the shear stress-strain plot of a unidirectional lamina, as seen in



Fig. 5. Predicted and experimental S-N curve for [30/-30]s carbon/epoxy T300/5208.



Fig. 6. Predicted and experimental S-N curve for [0/90/45/-45]s carbon/epoxy T300/5208.

Fig. 3. The error is measured by the  $\chi^2$  statistical measure of the difference between the predicted values  $p_i$  and the experimental values  $e_i$ . The values obtained are shown in Table 3.

T300/5208 with R = 0.1 [37], as shown in Fig. 4. The remaining parameters in Table 4 are obtained directly from experimental data [3,10,12,26,37].

The parameters  $\alpha$  and  $\beta$  given in Table 4 were adjusted to fit the S-N data of [45/-45]2s carbon/epoxy

Keeping all the parameters fixed, the S-N curve of T300/5208 [0/30/-30]6s laminate [26], also at R = 0.1, was predicted and compared with experimental data in Fig. 5.



Fig. 7. Effect of strain ratio R for [0/90/45/-45]s carbon/epoxy T300/5208.

Using the same parameters, the predicted and experimental S-N curves of T300/5208 [0/90/45/-45]s laminate [12], also at R = 0.1, are shown in Fig. 6.

The effect of the strain ratio R on the number of cycles to failure of the 90° lamina of the [0/90/45/-45]s laminate is shown in Fig. 7.

#### 5. Conclusions

The proposed model utilizes 11 parameters, which need to be adjusted with experimental data. Of these, nine are adjusted with quasi-static lamina strength data, which is much easier to obtain than fatigue data. Only two parameters are adjusted with fatigue data from a laminate. A fatigue damage evolution model using only two adjustable parameters was integrated with a quasistatic damage model to predict orthotropic damage. Therefore, the orthotropic nature of damage in PMC is accounted for. The nine constants of the quasi-static model are adjusted with available lamina strength data. Further validation with experimental data for other material systems would be desirable as the quasi-static and fatigue data for new material systems becomes available.

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# Appendix A

To compute the derivatives in Eq. (29), take a derivative of Eq. (2)

$$\frac{\partial M_{ijkl}^{r}}{\partial \Omega_{mn}^{r}} = -\left[\frac{\zeta_{ijkl}\langle\sigma_{kl}^{\prime}\rangle}{(1-\Omega_{ij}^{\prime})^{2}} + \frac{C_{n}\zeta_{ijkl}\langle-\sigma_{kl}^{\prime}\rangle}{(1-C_{n}\Omega_{ij}^{\prime})^{2}}\right]\frac{\partial\Omega_{ij}^{\prime}}{\partial\Omega_{mn}} \\ -\frac{1}{2}C_{s}(I_{ijkl}-\zeta_{ijkl})\left[\frac{1}{(1-C_{n}\Omega_{il}^{\prime})^{2}}\frac{\partial\Omega_{il}^{\prime}}{\partial\Omega_{mn}} + \frac{1}{(1-C_{n}\Omega_{jj}^{\prime})^{2}}\frac{\partial\Omega_{jj}^{\prime}}{\partial\Omega_{mn}}\right].$$
(A.1)

Since

$$\frac{\partial M_{ijkl}}{\partial \Omega_{pq}} \frac{\partial \Omega_{pq}}{\partial M_{ijkl}} = 1 \quad \text{(no summation)}, \tag{A.2}$$

$$\frac{\partial M_{r_{smn}}^{L}}{\partial M_{toxz}^{r}} = \frac{\partial M_{r_{smn}}^{L}}{\partial \Omega_{ij}^{L}} \frac{\partial \Omega_{ij}^{L}}{\partial \Omega_{pq}^{r}} \frac{\partial \Omega_{pq}^{r}}{\partial M_{toxz}^{r}} \quad (\text{no summation}), \quad (A.3)$$
$$r = f, m$$

now differentiate Eq. (4) with respect to  $M^{L}$  [8]

$$\frac{\partial M_{ijrs}}{\partial M_{mnpq}^{L}} = \left(c^{f} M_{ijkl}^{f} B_{kluv}^{f} + c^{m} M_{ijkl}^{m} B_{kluv}^{m}\right) J_{uvrs} + \left(c^{f} \frac{\partial M_{ijkl}^{f}}{\partial M_{mnpq}^{L}} B_{kluv}^{f} + c^{m} \frac{\partial M_{ijkl}^{m}}{\partial M_{mnpq}^{L}} B_{kluv}^{m}\right) M_{uvrs}^{L}$$
(A.4)

and similarly to Eq. (A.3) we get

$$\frac{\partial \Omega_{rs}}{\partial \Omega_{to}^{\rm L}} = \frac{\partial \Omega_{rs}}{\partial M_{ijxz}} \frac{\partial M_{ijxz}}{\partial M_{mnpq}^{\rm L}} \frac{\partial M_{mnpq}^{\rm L}}{\partial \Omega_{to}^{\rm L}},\tag{A.5}$$

which completes the derivation.

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