Equations facilitate composite designs

MODERN PLASTICS AWARD WINNER
BEST-OF-CONFERENCE
SPI COMPOSITES CONFERENCE 1999

BY PIZHONG QIAO*, JULIO F. DAVALOS*, EVER J. BARBERO*, DUSTIN TROUTMAN**

Useful guidelines based on key stress-factor equations benefit designers of FRP structures.

Plastics-based composite beams and columns can be used for bridges, piers, retaining walls, and storage structures exposed to salts and chemicals. However, the lack of step-by-step procedures for designing fiber-reinforced plastic (FRP) beams presents a problem to builders, engineers, and others who are unfamiliar with composites, yet bear the liability for making material choices.

We present a set of design equations for such structural members that account for bending, shear, local/global buckling, and material failure that are based on experimental and analytical study of eight commonly used FRP shapes. Together with our guidelines, they can be used by structural engineers in designing composite structures.

Most FRP shapes are thinwalled, manufactured by the pultrusion process, and typically consist of E-glass fiber and polyester or vinyl ester resins. The complex interaction of material properties, architecture, and geometric shapes requires that four key structural behaviors be considered in the design. These include:

- Relatively large deflections due to the low elastic modulus of resins used.
- Considerable shear deformation due to the relatively low shear modulus of the composite.
- Critical global and local stability (buckling) due to the thinwall structure and/or large slenderness ratios of component panels.
- Potential material failure due to the relatively low compressive and shear strengths of composites.

*West Virginia Univ., Morgantown, WV. **Creative Pultrusions Inc., Alum Bank, PA.

To address these issues, design equations based on parameters tabulated for 24 representative FRP sections produced by Creative Pultrusions Inc., Alum Bank, PA, were developed in this study, sponsored by the West Virginia Department of Highways. Key parameters include panel stiffness and strength, beam bending/shear stiffness, beam deflection and maximum strain, global critical buckling load, flange local critical buckling load, and beam bending and shear strength. Eight of the 24 beam sections were tested – four wide-flange beams, a box beam, two I-beams, and a channel beam.

The design parameters and their corresponding simplified step-by-step design equations for FRP beams are presented, and the accuracy of the equations is validated with experimental data. For most pultruded FRP sections, the layup of a panel is usually balanced symmetrically, and the panel stiffness and strength properties can be obtained either from experimental coupon tests or through theoretical predictions by micro/macromechanics. The panel stiffness and strength properties obtained from coupon tests compare well with predicted values. (Tables 1, 2) Explicit equations developed in terms of panel stiffness and strength properties can be applied in engineering design for the computation of beam bending and shear stiffness coefficients, deflections, panel strains and stresses, local/global buckling loads, and material failure loads.

Table 1. Panel stiffness properties of FRP shapes

FRP shapes	E_x ($\times 10^6$ psi) Tension test	Micro/macro-mechanics	G_{xy} ($\times 10^6$ psi) Iosipescu test	Micro/macro-mechanics
WF6x6x 3/8-in	4.155 (COV = 5.3%)	4.206	0.686 (COV = 8.4%)	0.682
I4x8x 3/8-in	5.037 (COV = 2.2%)	4.902	0.745 (COV = 9.8%)	0.794
WF4x4x 1/4-in	4.391 (COV = 5.6%)	4.167	0.778 (COV = 11.3%)	0.676
B4x4x 1/4-in	4.295 (COV = 10.7%)	3.604	0.548 (COV = 8.4%)	0.550

WF: wide flange beam, I: I-beam, B: box beam, COV: coefficient of variation

The response of FRP shapes in bending is evaluated using the mechanics of thinwalled laminated beams (MLB). We simplify the MLB formulations and present explicit expressions in terms of panel engineering properties for beam bending and shear stiffness coefficients, which can be used in equations for predicting beam deflections, and bending strains and stresses. Assuming that the beam centroid is the neutral axis of bending (no beam bending-extension coupling), general expressions for the beam bending (D) and shear stiffness (F) coefficients are computed (see Eq. 1) where b_i is the panel width, t_i is the panel thickness, and ϕ_i is the

Eq. 1

$$D=\sum_{i=l}^{n}\left[(E_x)_i t_i\left(h_i^{\,2}+\frac{b_i^2}{12}\sin^2\phi_i\right)+\frac{(E_x)_i t_i 3}{12}\cos^2\phi_i\right]b_i,$$

$$F=\sum_{i=l}^{n}(G_{xy})_i t_i b_i \sin^2\phi_i$$

Eq. 2

$$\delta=\delta_b+\delta_s=\frac{PL^3}{48D}+\frac{PL}{4K_\gamma F}$$

Eq. 3

$$\varepsilon_x=\frac{M}{D}h_i \; and \; \gamma_{xy} = \frac{V}{F}\sin\phi_i$$

Eq. 4

$$\sigma_x^{\,cr}=\frac{\pi^2}{12}\left(\frac{t_f}{b}\right)^2\left[\sqrt{q}\left(2\sqrt{(E_x)_f(E_y)_f}\right)+p\left((E_y)_f(v_{xy})_f+2(G_{xy})_f\right.\right.$$

$$"I": p=0.3+\frac{0.004}{\zeta-0.5};q=0.025+\frac{0.065}{\zeta+0.4};\zeta=\frac{2b_w}{b_f}\frac{(E_y)_f}{(E_y)_w};\; b=\frac{b_f}{2}$$

$$Box: p=2.0+\frac{0.002}{\zeta-1.3};q=1.0+\frac{0.08}{\zeta+0.2};\zeta=\frac{b_w}{b_f}\frac{(E_y)_f}{(E_y)_w};b=b_f$$

Eq. 5

$$P_{cr}^{\,local}=\frac{8D\sigma_x^{\,cr}}{(E_x)_f b_w L}$$

Eq. 6

$$P_{cr}^{\,global}=\frac{17.17}{L^2}\sqrt{D\cdot JG}\sqrt{1+\frac{\pi^2}{L^2}\frac{I_{ww}}{JG}} \text{ where}$$

$$JG=\frac{2(G_{xy})_f t_f^{\,3} b_f}{3}+\frac{(G_{xy})_w t_w^{\,3} b_w}{3}; \text{ and}$$

$$I_{ww}=\frac{(E_x)_f t_f b_w^{\,2} b_f^{\,3}}{24}+\frac{(E_x)_f t_f^{\,3} b_f^{\,3}}{36}+\frac{(E_x)_w t_w^{\,3} b_w^{\,3}}{144}$$

Eq. 7

$$Bending: P_{fail}^{\,bending}=\frac{8F_c D}{(E_x)_f(b_w-t)L};\; Shear: P_{fail}^{\,shear}=F_{xy}b_w t_w$$

cross-sectional orientation of the *i*th panel with respect to the bending axis; $(E_x)_i$ and $(G_{xy})_i$ are the panel stiffness values obtained either by the micro/macromechanics approach or from coupon tests. Since D and F are similar to EI and GA for isotropic materials (e.g., steel beams), if the flanges and webs of a section have identical layups and stiffnesses, then the beam bending and shear stiffnesses can be expressed in terms of panel stiffnesses E_x and G_{xy} and geometric properties I and A.

Displacement and rotation functions can be obtained by solving Timoshenko's beam theory equilibrium equations. In particular, available expressions for maximum bending and shear deflections can be used. For example, the maximum deflection for a three-point loading of a beam of span L and design load P is shown where the bending (δ_b) and shear (δ_s) components of deflection can be independently evaluated. As an approximation in design, the shear correction factor for most FRP sections can be taken as K_γ = 1.0. (see Eq. 2) The maximum top-surface longitudinal strains and in-plane shear strains of the *i*th panel are expressed (Eq. 3) where V and M are, respectively, the resultant internal shear force and bending moment acting on the beam; h_i is the transverse coordinate of a point from the neutral axis.

A comprehensive analytical approach was developed to study the local buckling behaviors of pultruded FRP shapes. The local buckling analysis for discrete laminat-

Table 2. Panel strength properties of FRP shapes

FRP shapes	F_c ($\times 10^3$ psi) Compression test	Strength	F_{xy} ($\times 10^3$ psi) Iosipescu test
WF6x6x3/8-in	54.498 COV = 3.76%	45.55	12.866 COV = 2.36%)
I4x8x3/8-in	61.060 COV = 2.41%	56.65	13.022 COV = 8.13%
WF4x4x1/4-in	57.133 COV = 3.50%	53.10	13.167 COV = 29.17%
B4x4x 1/4-in	60.657 COV = 5.33%	47.20	11.138 COV = 4.84%

WF: wide flange beam, I: I-beam, B: box beam, COV: coefficient of variation

ed plates or panels of FRP shapes was formulated, and the effects of restraint at the flange-web connection were considered. For the flange panels under compression, simplified expressions for predictions of plate buckling strength are proposed by approximately solving transcendental equations where σ_x is the critical stress, and p and q are constants that are defined by the coefficient of restraint (ζ) at the junction of panels. (Eq. 4)

For a beam under three-point bending, the critical local buckling load (P_{cr}^{local}) can be obtained in terms of critical stress and beam properties. (Eq. 5). Results based on design equations 4 and 5 compare favorably with testing data for four wide-flange beams.

Global buckling in long-span FRP beams without lateral supports and with large slenderness ratios is common. We derived a Vlasov theory-based simplified equation for flexural-torsional buckling of an *I*-section (Eq. 6) which also correlates with experimental tests.

Due to relatively low compressive and shear strength of FRP composites, material failure must also be evaluated. Just as with deflection and buckling, beam bending and shear strength (ultimate failure loads) can be expressed in terms of panel strength properties (Eq. 8). Values for F_c and F_{xy} are in Table 2. The comparisons between the design based on experimental coupon data are also shown in Table 2.

Design procedures for FRP beams

The following guidelines for designing FRP beams under bending are recommended:

- Characterize the beam panel material properties (stiffness and strength) from either coupon tests or micro/macromechanics and empirical formulas.
- From Eq. 1, obtain the beam bending and shear stiffness coefficients, which in turn can be used to predict the beam deflection, strains, and stresses.
- Determine the local and global buckling resistance of beam sections by Equations 5 and 6, respectively.
- Predict the beam failure (bending and shear) loads based on the panel strength data and Eq. 7.

The simplified parameters and equations developed and presented here account for most of the critical issues in FRP beam design. They can be used in the future to develop general design guidelines and also "product-acceptance" criteria for FRP beams produced by any manufacturer. *–Edited by William A. Kaplan* ❑

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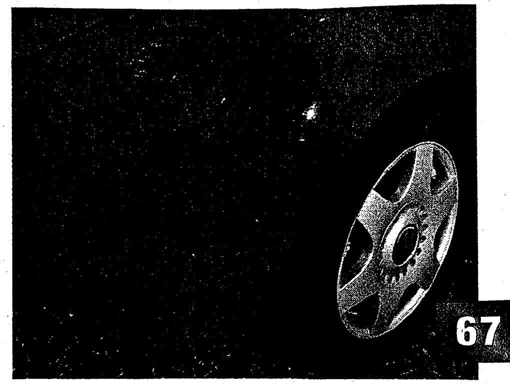
NOVEMBER 1999



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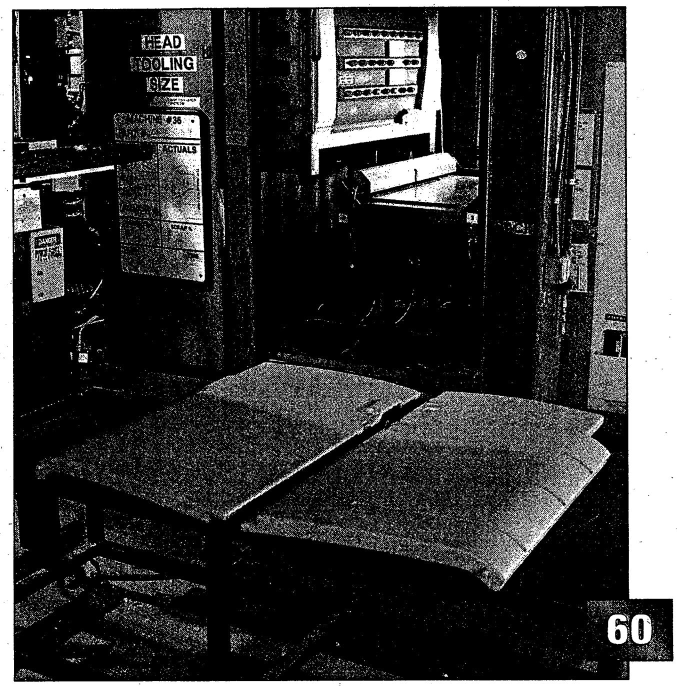

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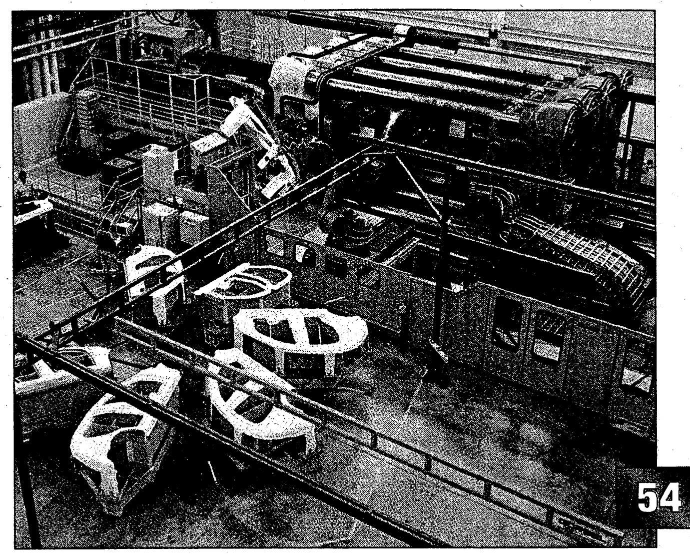

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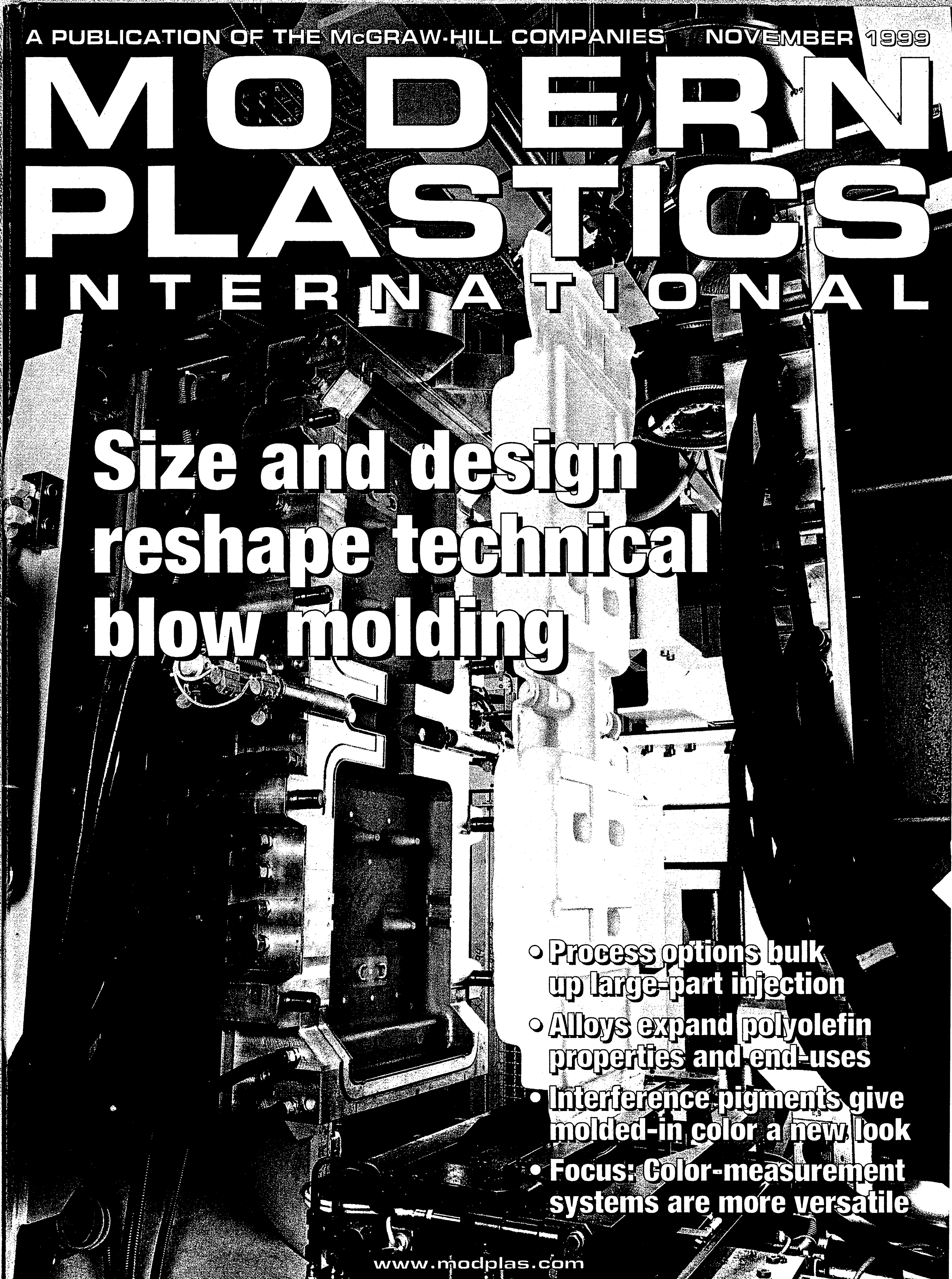

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