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John S. Tomblin[1] *and Ever J. Barbero*[2]

Statistical Microbuckling Propagation Model for Compressive Strength Prediction of Fiber-Reinforced Composites

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ABSTRACT: Compressive strength prediction for a fiber-reinforced composite material still remains an unresolved topic when dealing with composites in the design process. Although significant scatter is present in the experimental data, experimental test results are the only criteria on which to base design parameters. Although significant advances have been accomplished recently by various modeling techniques, only quantitative comparison with experimental data may be realized. This quantitative comparison requires the use of a semi-empirical parameter into the model formulation, which is usually set as the fiber misalignment. By using a single value of the fiber misalignment within the composite, model predictions easily match the experimental data because of the extreme sensitivity of the model with respect to fiber misalignment. However, it is well known that there is not a unique value of fiber misalignment for all the fibers but rather a distribution of misalignments throughout the composite. In this paper, using the complete fiber misalignment distribution, stability theory is coupled with continuous damage mechanics to generate a model for compressive strength of continuous fiber-reinforced composites. Sample results are also presented showing the correlation of the analytical model with experimentally measured strengths.

KEYWORDS: compression strength, fiber misalignment, microbuckling, elastic stability

The application of composites in advanced structures is gradually increasing as more information about their mechanical properties, manufacturing techniques, corrosion resistance, stealth characteristics, and durability in various operating environments becomes available. Current design requirements mandate the use of composites in various areas primarily because of the greater strength-to-weight and stiffness-to-weight ratios over conventional metallic materials. As the use of composites continues to grow, it becomes necessary to be able to predict failure of various structural components in order that appropriate factors of safety can be applied to the designs. Unfortunately, predicting and determining one of the primary material properties (the compression strength) still remains an unsolved subject in the behavior of composites.

The major difficulty with the current models for the compressive strength evolves around the fiber misalignment (on the order of 1 to 2°) existing within the cured composite. It has long been known and hypothesized that fiber alignment is extremely critical with respect to compressive strength since the misaligned fibers are in a buckled state initially. Since a

[1]Assistant professor, Department of Aerospace Engineering, Wichita State University, Wichita, KS 67260-0044.

[2]Associate professor, Department of Mechanical & Aerospace Engineering, West Virginia University, Morgantown, WV 26506-6106.

distribution of misalignment exists within the composite, a model with an idealized single fiber misalignment value (which represents the misalignment within the entire composite) becomes invalid. Because of the high sensitivity of the predicted compression strength to the fiber misalignment, virtually any experimental compressive strength value may be matched with the theoretical model predictions by choosing the value of the fiber misalignment. Hence, any gains realized by theoretical model predictions are immediately offset by arbitrarily choosing or guessing a value of existing fiber misalignment within the cured composite.

In this paper, a classical damage mechanics formulation is used to incorporate a Gaussian fiber misalignment distribution into an imperfection sensitivity model for compressive strength. The resulting model provides a deterministic, reliable value for predicting composite compressive strength, which may be used for design purposes based on misalignment distribution data obtained for each class of advanced composites.

Background

Compression strength of continuous fiber-reinforced composites is one of the most important material properties that strongly influences the design and utility of composites for structural applications. After three decades of research, models for the failure of unidirectional composites are just starting to appear that show reasonable comparison with experimental results [*1–5*]. Although these models have made significant improvements towards the prediction and understanding of compression strength, comparison is accomplished by introducing a semi-empirical parameter into the formulation that is chosen based upon the experimental compressive strength results. Hence, agreement between the experimental data and the analytical models is achieved by introducing an appropriate knockdown factor (the fiber misalignment value) into the formulation.

By using past models for compressive strength [*1–5*], it has been shown that the compressive behavior of continuous fiber composites is strongly influenced by several factors. Two factors believed to dominate the compressive response are the nonlinear shear stress-strain behavior and the fiber misalignment or waviness. Unfortunately, these two factors are synergistic and act in unison to drastically degrade the compressive strength of fiber-reinforced composites. Hence, the accurate characterization of these material properties is essential to any model validation.

Unidirectional continuous fiber composites have small angular deviations (between 1 to 10°) about their mean direction even though all fibers may be aligned in the same nominal direction. The detrimental effect of fiber misalignment has been shown to effect the compressive strength experimentally [*6,7*]. By taking measurements of the fiber misalignment existing within the cured composite [*8*], *quantitative* comparisons may be realized between the compressive strength and fiber misalignment distributions by using the mean or standard deviation of the measurements.

Currently, there are several compressive strength models that appear to predict the compressive strength relatively well [*1–4*]. These models have accounted for such effects as the nonlinear shear response of the matrix, initial fiber misalignment, shear deformation of the fibers, and random fiber spacings [*3,4*]. Since a variety of misalignment magnitudes exist within the composite, simply choosing a single representative value of the fiber misalignment only quantitatively verifies the model predictions. Given the extreme sensitivity of these compressive strength models with respect to fiber misalignment values, the exact nature as to the true effect the entire fiber misalignment distribution has with respect to the compressive strength remains unresolved.

To compound the problem further, not only does the fiber misalignment distribution vary with respect to the fiber and resin system used, but also with respect to the manufacturing method. Thus, different experimental strength results are likely to be obtained for the same fiber and resin system if manufactured by different methods, even when the same test fixture is used. This may be one of the main reasons such a wide scatter band is present in the compressive strength data available in the literature.

Nonlinear Shear Characterization

Previous models relating the compressive strength have shown the importance of including the nonlinear shear response [*1–5*]. In fact, if the nonlinear shear response is removed from the formulation, the postbuckling response of the system becomes stable and imperfections in the form of fiber misalignments are not expected to lower the compressive strength [*9,10*]. Hence, it becomes very important to have an accurate and reliable characterization of the nonlinear shear response. Unfortunately, the shear response is also one of the most difficult material properties to characterize.

The exact role the fiber misalignment has with respect to the shear response is unknown. From the basis of coordinate transformation, the misalignment should not affect the compressive strength until the fibers are misaligned at approximately 10° off-axis for a linear analysis. But the effect the misalignment has with respect to the complete nonlinear curve has not been investigated. Therefore, by characterizing the shear response on a specimen produced by the same manufacturing method and layup configuration of the specimens for which the fiber misalignment and compressive strength was determined, a meaningful comparison between model and experiments may be accomplished.

Problems in measuring the nonlinear shear response are still encountered at present day even though ASTM standards exist. Various methods for the shear response characterization include the [±45] coupon test, the 10° off-axis test, the rail shear, Iosipescu, Arcan, and the torsion test. Although all these methods produce relatively acceptable values of the linear shear modulus, wide scatter exists in the nonlinear region and the ultimate shear strength [*11*].

An additional problem encountered in the shear response is modeling the behavior for input into an analytical model. Since classical rule-of-mixtures formulas are not accurate for the shear response, it becomes necessary to seek a reliable relationship to represent the nonlinear shear stress-strain diagram. In the current investigation and for the purposes of developing a compressive strength model, the following relationship for modeling the shear response was assumed

$$\tau(\gamma) = \tau_{\mathrm{ULT}} \tanh\left(\frac{G_{\mathrm{LT}}\gamma}{\tau_{\mathrm{ULT}}}\right) \tag{1}$$

where τ_{ult} is the shear strength, and G_{LT} is the longitudinal shear modulus. This hyperbolic relationship has the proper characteristics for large strains and is asymmetric with respect to the origin, which is important to produce a symmetric bifurcation point in the stability analysis. In addition, this equation depends only upon two constants, which are typically measured and reported for shear responses. Figure 1 shows the correlation between the hyperbolic relationship with respect to two experimentally obtained shear responses for a unidirectional graphite/epoxy [AS4/E7K8] composite. The data reported in Fig. 1 were generated by a 0° Iosipescu specimen (90° notch) and have been corrected according to the procedure described in Ref *11*. For the compressive strength model formulation proposed, an accurate and reliable nonlinear relationship between shear stress and strain is essential for model validation.

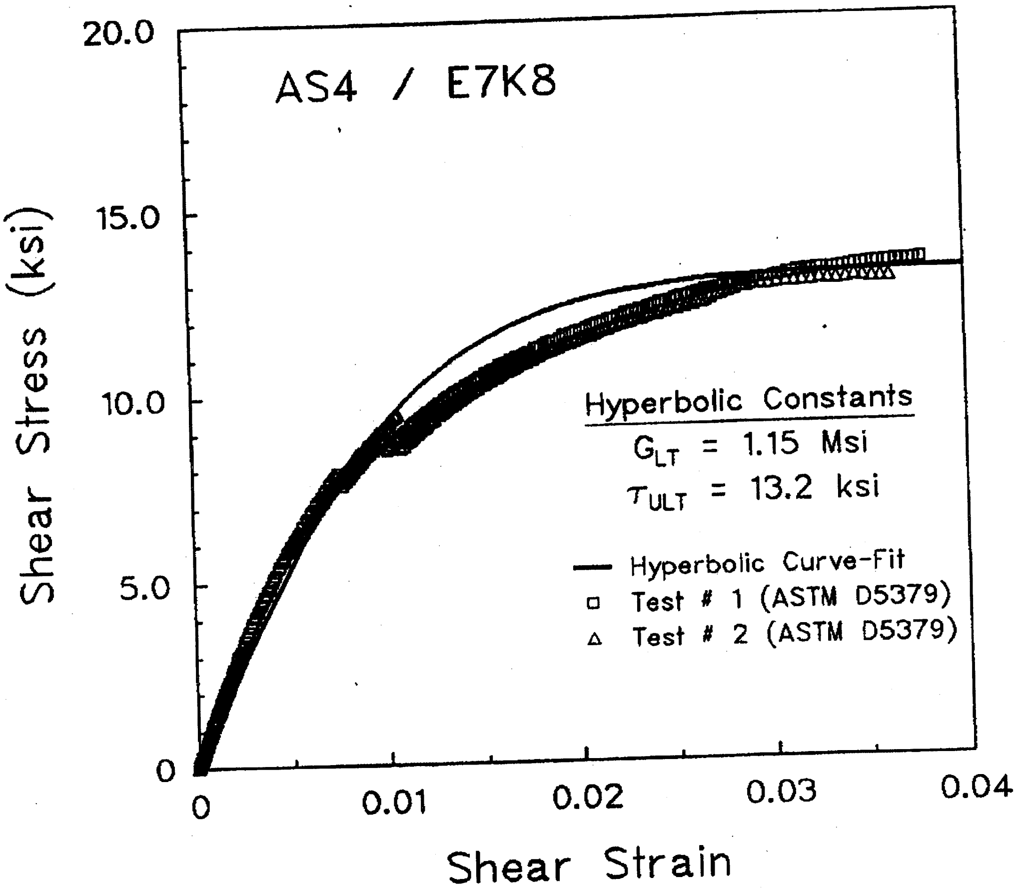


FIG. 1—*Typical shear response (AS4/E7K8) along with hyperbolic curve-fit.*

Stability Model for Compressive Strength

Buckling of perfectly aligned fibers was considered initially by Rosen [*12*] and has since been modified by many investigators. Although Rosen's formulation produced unrealistic values of the compressive strength, the fundamental assumptions, such as shear mode microbuckling and the two-dimensional fiber-matrix model, still prove useful in the current formulations. Presently, by modifying Rosen's initial formulation accounting for nonlinear shear response and fiber misalignment, more realistic values of compressive strength are obtained [2,9,*10*].

Using these fundamental assumptions in an energy approach, the authors have also established a two-dimensional compressive strength model formulation using the generalized theory of elastic stability [*3*] based upon the representative volume element shown in Fig. 2 and assuming the fibers to fail in the shear microbuckling mode. The results obtained from this model are similar to other current formulations in that the compressive strength is dominated by the nonlinearity of the shear response and the initial fiber misalignment. As shown in Refs *9* and *10*, an analysis of the perfect system (in which no fiber misalignment exists) yields an unstable, symmetric bifurcation point at

$$\sigma_{CR} = G_{LT} \tag{2}$$

which is exactly the bifurcation load originally obtained by Rosen [*12*]. By using the stability formulation, the postbuckling nature of the bifurcation point was established as *unstable.* This result indicates that imperfections, in the form of fiber misalignments, are

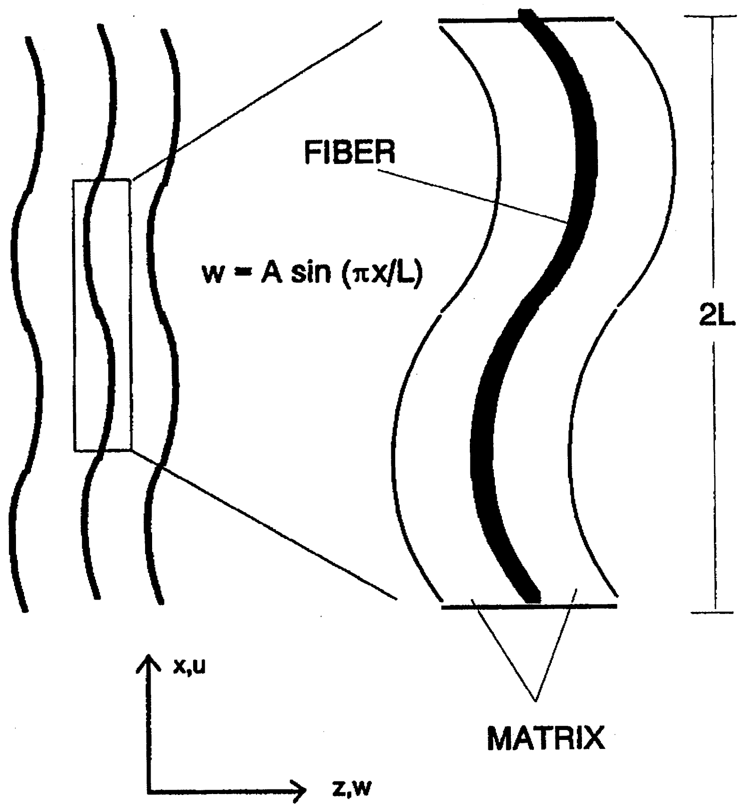
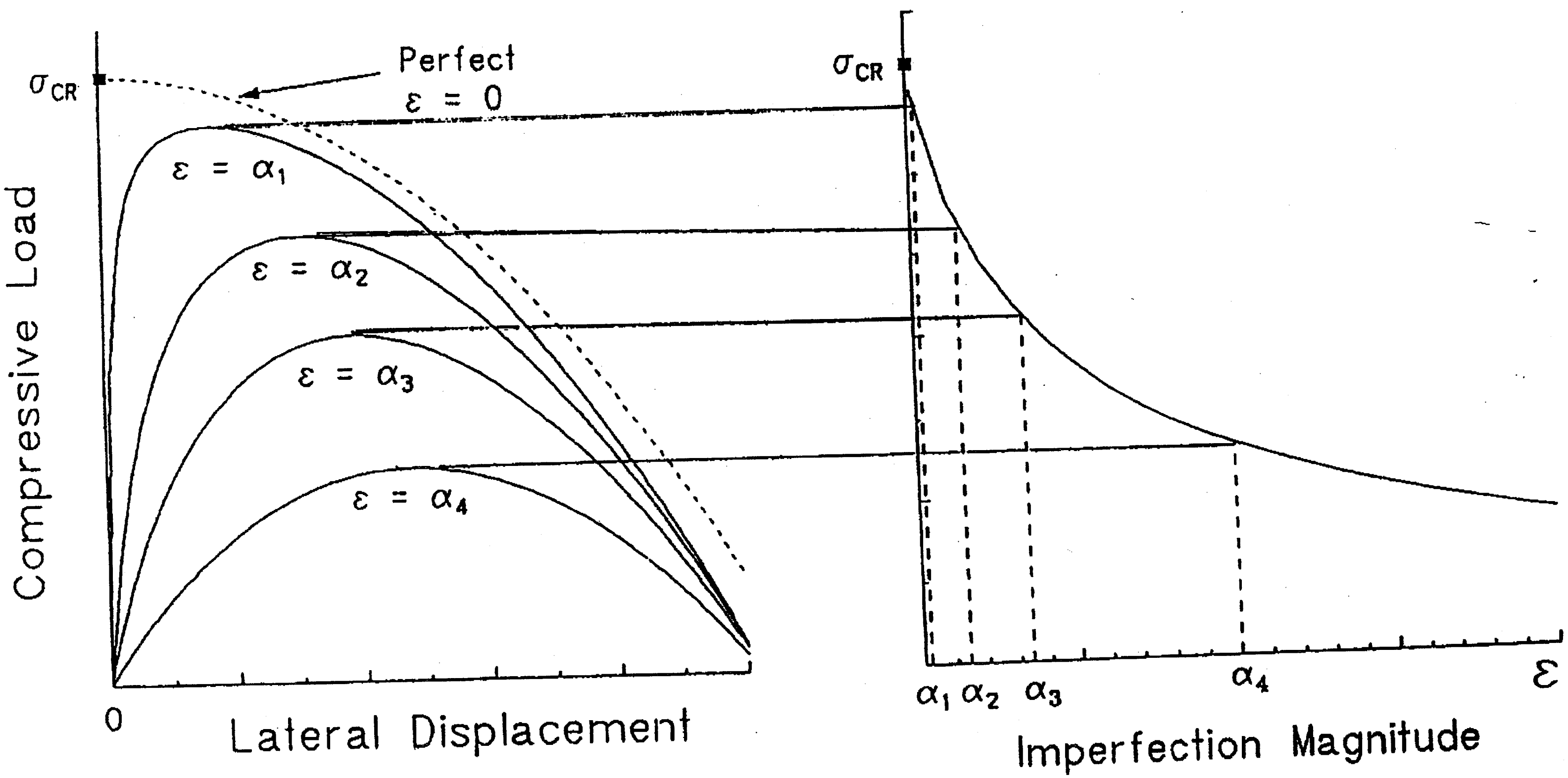


FIG. 2—*Representative volume element used for stability model.*

expected to lower the maximum obtainable compressive load in the composite (this result is similar to the postbuckling response of axially loaded cylindrical shells). This unstable phenomenon is depicted in Fig. 3*a* for varying degrees of imperfections (fiber misalignment). Hence, once the loading of the fibers reaches the apex of the equilibrium path, the load-carrying ability of that specific fiber is drastically reduced. The details of the stability formulation are beyond the scope of this paper and may be obtained from references [*9,10*]. Models similar to the one proposed have also been presented by other authors using different formulations [*1–4*].

Using the stability model formulation, the postbuckling characteristics of the fibers, when coupled with the nonlinearity of the matrix in shear, produce an imperfection sensitive system in which the fibers have no postbuckling strength. As shown in Fig. 3*a*, the unstable nature of the postbuckling path produces a limit load condition in which the bifurcation point traditionally encountered in buckling problems cannot be obtained, thereby lowering the maximum attainable compressive load. As shown in Fig. 3*a*, for increasing imperfections ϵ (i.e., the fiber misalignment) the maximum attainable compressive load within the composite is reduced. By connecting these peaks for increasing values of fiber misalignment, an imperfection sensitivity curve may be established as depicted in Fig. 3*b*.

This imperfection sensitivity relationship is extremely important to the validation of compression strength model formulations. From the stability viewpoint, this curve is customarily produced using pertubation methods [*13*]. Due to the complexity of the analysis and the requirement to include large angles of misalignment (on the order of 3 to 6°), a closed form solution of this relationship is not possible. Hence, numerical methods must be employed to obtain an implicit relationship of the imperfection sensitivity curve. Figure 4 shows the imperfection sensitivity relationship using perturbation methods (up to the fifth order) along with the numerical solution. As seen from Fig. 4, perturbation methods achieve



(A)

(B)

FIG. 3—*Imperfection sensitivity curve generation for varying degrees of imperfection from a limit load condition.*

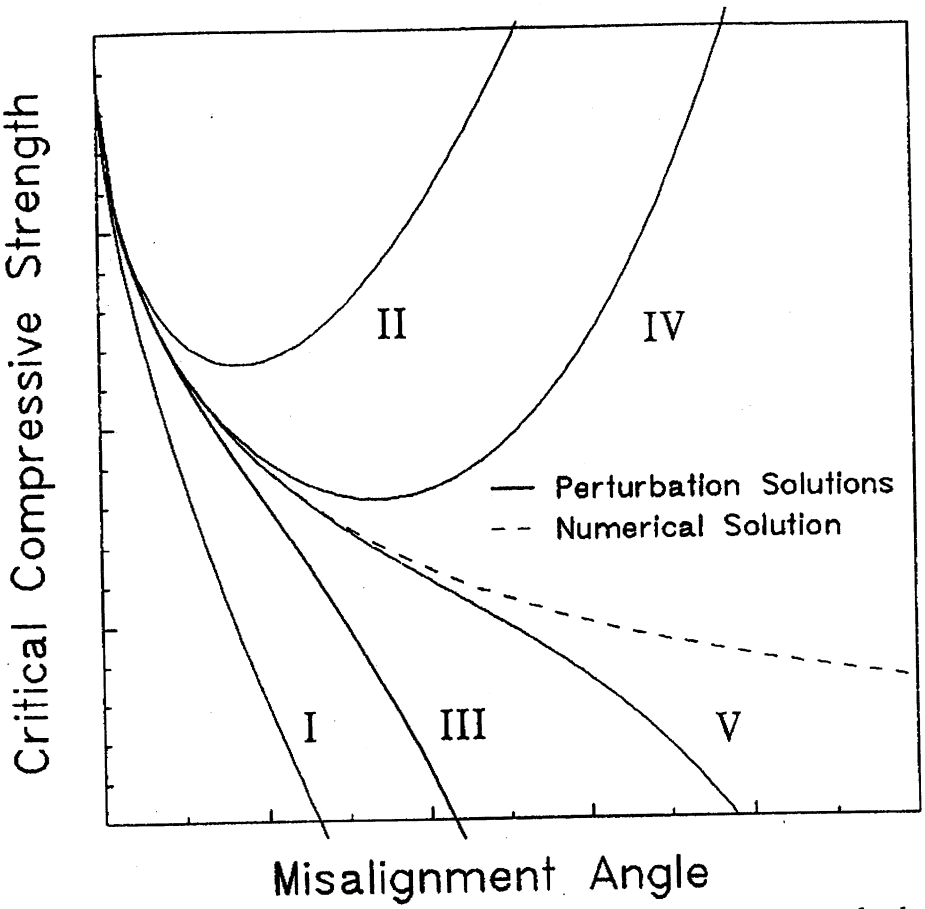


FIG. 4—*Imperfection sensitivity relationship using perturbation methods (up to fifth order) along with the numerical solution.*

the required degree of accuracy for very small misalignments but diverge at large magnitudes. Therefore, the use of perturbation methods in the analysis becomes unacceptable and the numerical solution must be adopted.

Since the primary goal of this paper is to establish a compressive strength model that couples together the complete distribution of fiber misalignment along with the stability characteristics of the misaligned fibers, it becomes necessary to establish an explicit relationship for the imperfection sensitivity curve. An exponential equation was proposed to fit the imperfection sensitivity curve in the form

$$\sigma_{CR}(\alpha) = (G_{LT} - k_1)e^{-k_2\alpha^{1/2}} + k_1 e^{-k_3\alpha^{1/2}} \tag{3}$$

where G_{LT} is the composite shear stiffness, α is the misalignment angle, and k_1, k_2, k_3 are functional constants determined by a curve fit. Note that Eq 3 has the following characteristics:

1. When $\alpha = 0$, $\sigma_{CR} = G_{LT}$ (Rosen's model).
2. When $\alpha \rightarrow \pi/2$, $\sigma_{CR} \rightarrow 0$.

Figure 5 shows a typical implicit numerical solution coupled with the curve fit using Eq 3. The curve-fit constants were obtained by minimizing the chi-squared test statistic [*14*].

Although the imperfection sensitivity curve enables one to quantitatively make a compari-

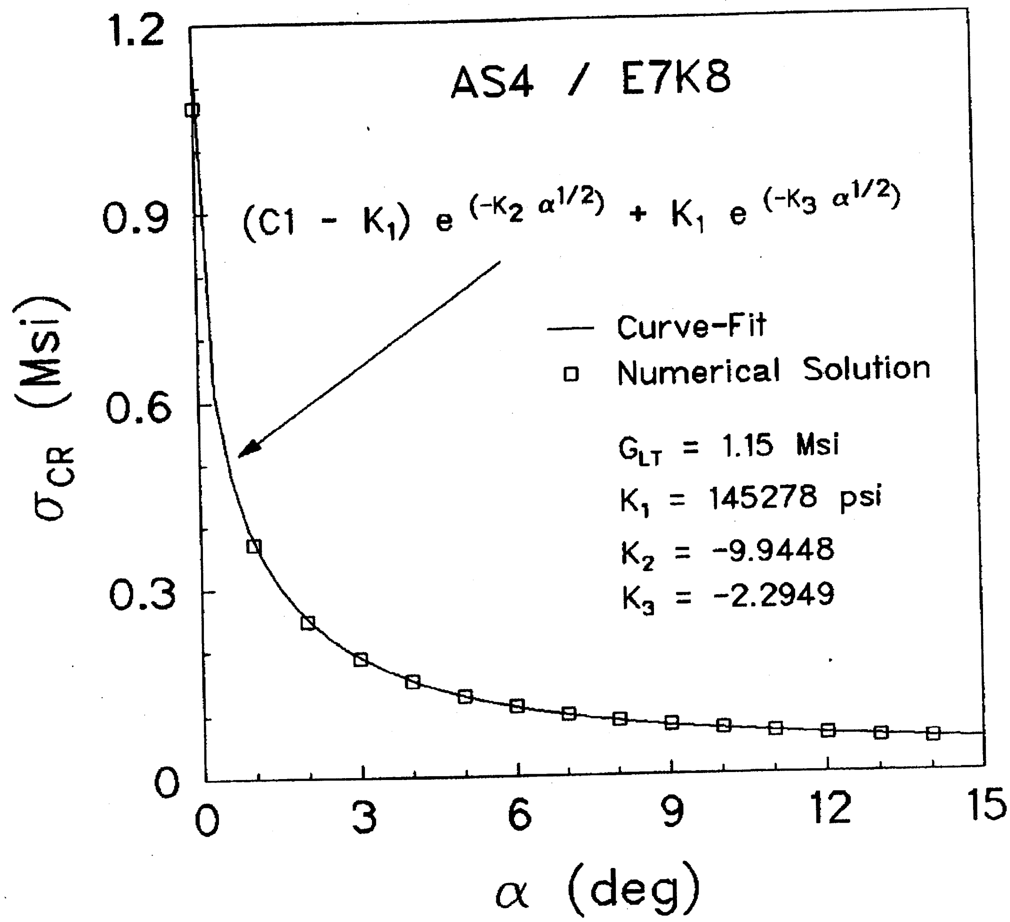


FIG. 5—*Numerical imperfection sensitivity relationship along with curve-fit equation (Eq 3).*

son for various types of fiber misalignment values, the model still encounters the same difficulties as other model formulations by using a representative value for the fiber misalignment. This effect has been counteracted by other investigators by using a value representative of the distribution, like the standard deviation, two times the standard deviation, expected value, etc. Furthermore, there is no theoretical justification for assuming all the fibers to have the same misalignment angle. Using this result as a basis for this investigation, the authors have proposed a more systematic approach to include the effect of the entire distribution into the formulation.

Fiber Misalignment Characterization and Modeling

Although measurement techniques have been established for characterizing the fiber misalignment, actual misalignment measurements for fiber-reinforced composites are sparse and seldomly reported. Due to the sensitivity of compressive strength with respect to fiber alignment, this commonly overlooked or hypothesized number or distribution becomes extremely important in verifying an analytical model.

Using the method developed by Yurgartis [8], misalignments within the cured composite may be obtained by sectioning the composite at a known angle Φ (this angle may also be defined by the mean misalignment distribution). Once sectioned, the cylindrical fibers form ellipses and may be observed microscopically by polishing the cut face. Figure 6 shows a typical cut section obtained for a unidirectional graphite-epoxy [AS4/E7K8] composite. By

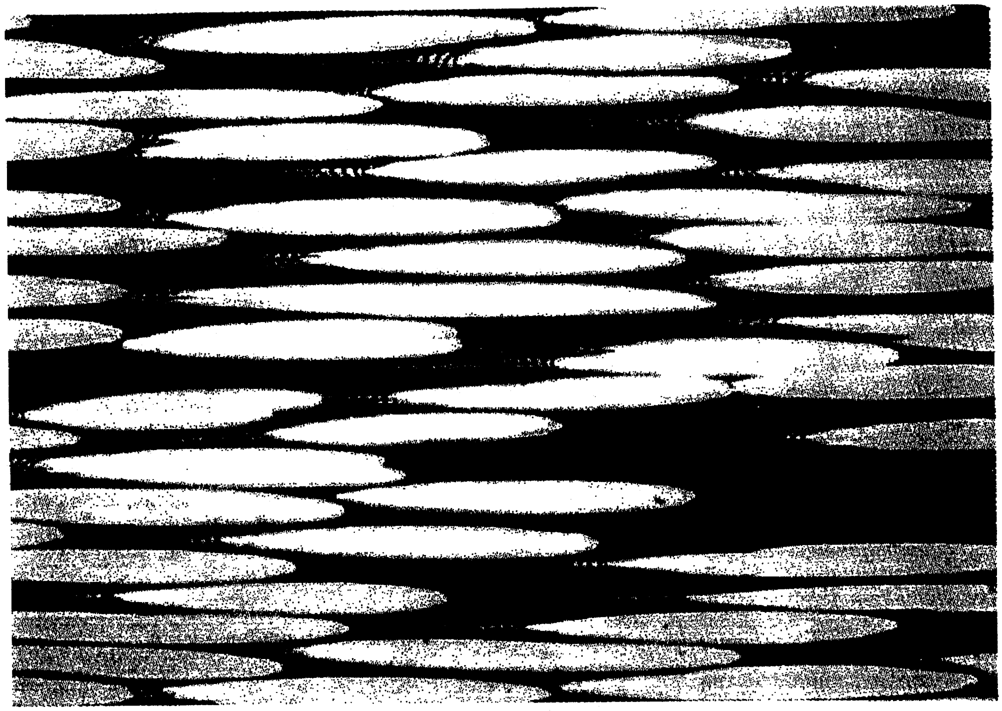


FIG. 6—*Typical microscopic field-of-view showing used for fiber misalignment measurements (AS4/E7K8).*

measuring the major and minor axis (fiber diameter) of the ellipses, the angle of that individual fiber with respect to the cut face ω may be obtained by the relation

$$\sin \omega = \frac{\text{minor axis length}}{\text{major axis length}} \tag{4}$$

By taking measurements randomly over the entire cross section, a distribution of angles may be obtained ω_i. Knowing the angle of cut Φ, the distribution with respect to the longitudinal direction may be obtained by

$$\alpha_i = \omega_i - \Phi \tag{5}$$

Since the fibers in a composite have a distribution of misalignment, the statistical nature of the misalignment must be incorporated into the compression strength model. Using the discrete transformed distributions, the next step in the statistical process is to chose a continuous distribution to represent the fiscal system. Once a continuous distribution is chosen, the statistical nature of the fiber misalignments may be incorporated into the analytical model.

Although the fiber misalignment data presented in the literature are sparse, the obvious choice for the statistical model is Guassian or normal with a probability function $f(x)$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(x-\mu)}{2\sigma^2}\right]; \ -\infty < x < \infty \tag{6}$$

where σ is the standard deviation, μ is the mean, and x is the continuous random variable. Assuming the distribution of the misalignment to be symmetric ($\mu = 0$), measurement bias and distortion of the measurement technique may be circumvented by discarding the "lower half" of the discrete transformed measurements and using the "upper half" to estimate the distribution $f(\alpha)$ [8]. The assumption of normality may also be checked using the cumulative distribution function (CDF) and probability scales [14].

Since fiber microbuckling occurs at the same load for positive or negative misalignment angles, it was necessary to convert the normal distribution to a "half" normal, which is the special case of the more general "folded" normal distribution [15,16]. In the "half" normal distribution, the random variable z is given as

$$z = |x| \tag{7}$$

where x is the random variable of the regular normal distribution. In other words, the "half" normal distribution represents the normal distribution without the algebraic sign. Hence, in application of the "half" normal distribution to the problem of compressive strength, positive and negative angles of misalignment have the same effect on compressive instability.

Using the new random variable z, the density of the "half" normal distribution is derived as [15,16],

$$f(z) = \frac{1}{\sigma}\sqrt{\frac{2}{\pi}}\exp\left[\frac{-z^2}{2\sigma^2}\right]; \quad z \geq 0 \tag{8}$$

or in terms of an misalignment angle α

$$f(\alpha) = \frac{1}{\sigma_\alpha}\sqrt{\frac{2}{\pi}}\exp\left[\frac{-\alpha^2}{2\sigma_\alpha^2}\right]; \quad \alpha \geq 0 \tag{9}$$

where σ_α is the standard deviation of the measured discrete normal distribution. Figure 7 shows the resulting density function of the "half" normal distribution. Physically, Eq 9 represents the probability that a fiber chosen at random in the cross section has a misalignment of value α. But more importantly, assuming the number of fibers in the cross section is large, Eq 9 gives the ratio of the number of fibers that have a misalignment α over the total number of fibers.

Damage Model Formulation

Based on the stability model presented previously [9,10], it was demonstrated that due to the unstable nature of the bifurcation, the fibers were unable to carry a postbuckling load. When this result was related to an imperfect system that contained fiber misalignments, the system became characterized by a limit load condition, the degree to which was determined from the initial misalignment of the fibers. Hence, considering an individual fiber with initial misalignment, once the critical load limit is reached, the fibers' ability to carry load is lost. The inability to carry the post-buckling load does not imply permanent damage within the composite once the limit load condition is reached. This result just indicates that the load-carrying capacity of that fiber is much lower than the applied load. The fiber may break in a bending mode when this lateral deflection becomes large [1,17].

From the basis of the inability of the fiber to carry a post-buckling load, a one-dimensional damage mechanics formulation [18] was proposed to account for the accumulation of the microbuckling process. According to the imperfection sensitivity curve, those fibers that

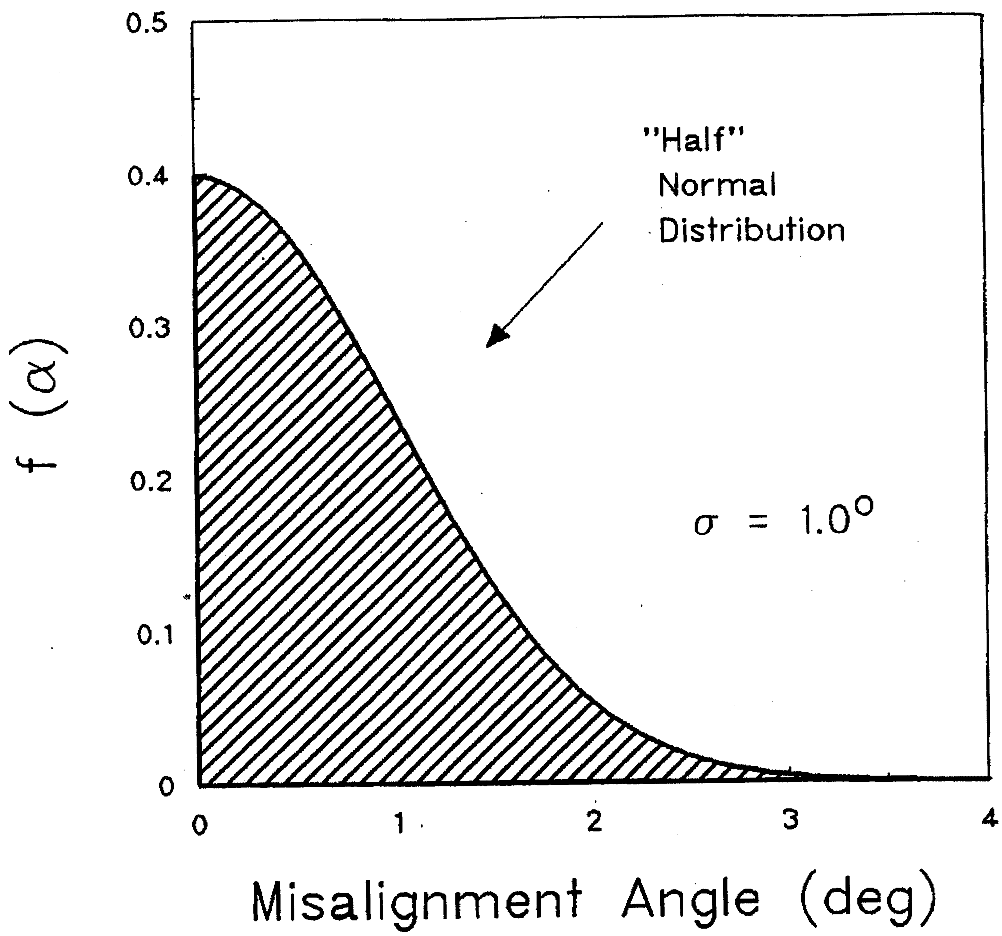


FIG. 7—*Probability density function for half-normal distribution.*

have a large initial misalignment value have a lower critical load limit. Therefore, once the critical load is reached, the load must be redistributed onto the remaining fibers that have not reached their limit load. The one-dimensional damage mechanics relation that describes this is given by the relation

$$\sigma_{max}(\alpha) = \sigma_{CR}(\alpha)[1 - \omega(\alpha)] \tag{10}$$

where $\omega(\alpha)$ is the area of failed fibers per unit of initial fiber area. The critical stress $\sigma_{CR}(\alpha)$ represents the stress at which the corresponding fiber has reached its limit load and is given by the imperfection sensitivity relationship (Eq 3). This failure is depicted graphically in Fig. 8. As seen from Fig. 8, the damage model induces the compressive failure process to proceed from high to low misalignment magnitudes.

Using the above relation, the expected compressive strength for a given composite may be obtained using the misalignment distribution. Unlike the previous models for compressive strength, the proposed model will account for the complete distribution of fiber misalignment values. Assuming the fibers may be modeled statistically using a probability density function $f(\alpha)$, the damage variable $\omega(\alpha)$ becomes

$$\omega(\alpha) = 1 - F(\alpha) = 1 - \int_0^\alpha f(\alpha)\,d\alpha = \int_\alpha^\infty f(\alpha)\,d\alpha \tag{11}$$

where $F(\alpha)$ is the cumulative density function representing the percentage of unfailed fibers. Figure 9 depicts the failure process with respect to the total number of fibers within the composite.

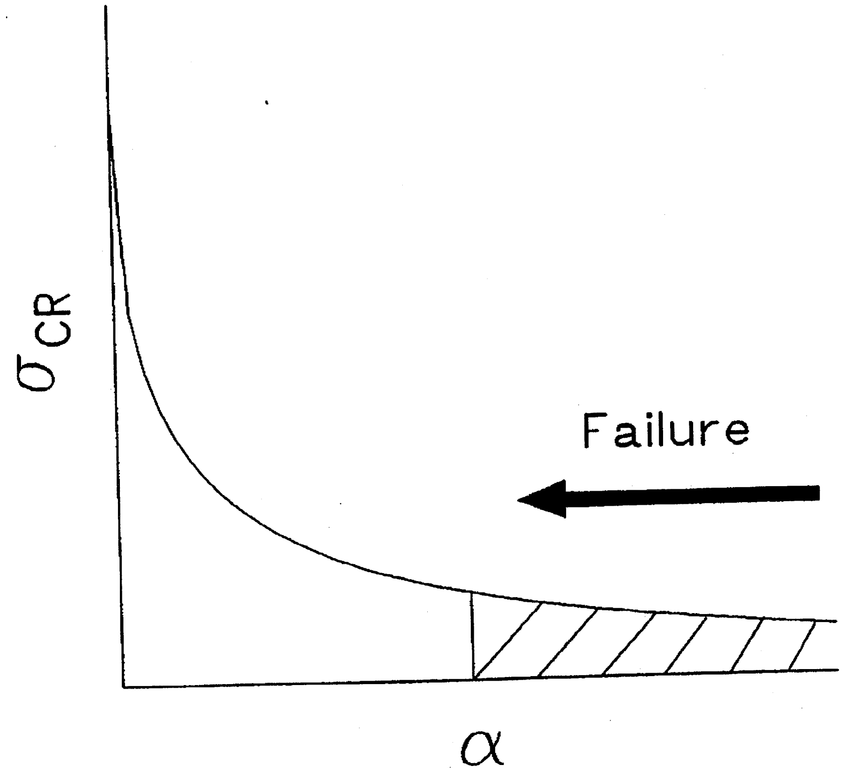


FIG. 8—*Failure propagation with respect to imperfection sensitivity curve.*

When the probability relationship is combined together with the one-dimensional damage mechanics law, a maximum is reached on the applied stress σ_{max} (Eq 10). Physically, this maximum is the point at which the compressive force overcomes the reloading process (see Fig. 10). This apex may be obtained by evaluating the derivative of the one-dimensional damage mechanics law (Eq 10) to obtain the corresponding critical misalignment angle α_{CR}. This value may then be substituted back into the governing equation to obtain the expected

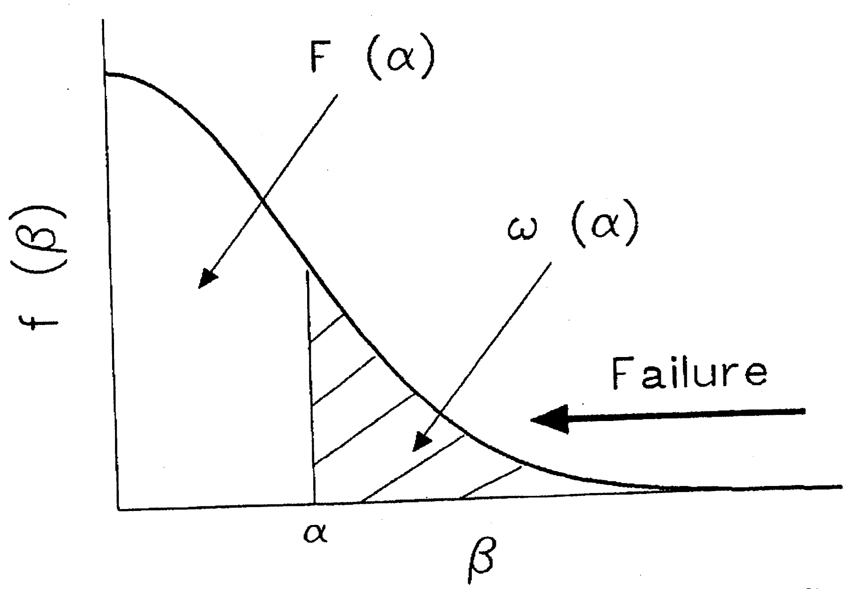


FIG. 9—*Failure propagation with respect to fiber misalignment distribution.*

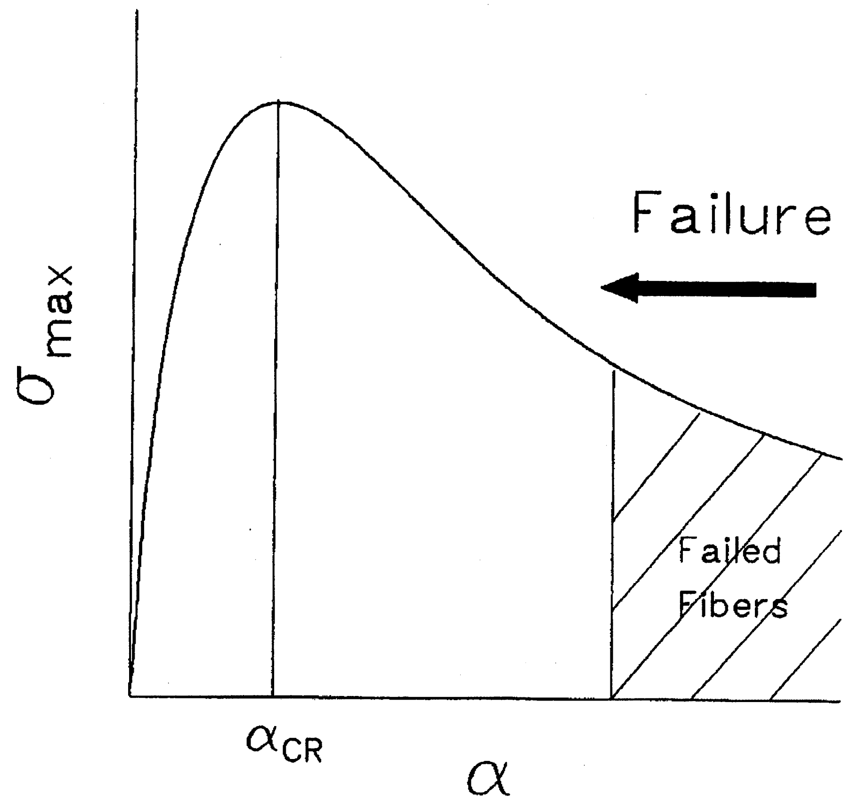


FIG. 10—*Maximum applied stress as a function of misalignment angle.*

value of the compressive strength. Figure 11 shows the solution procedure for obtaining the compressive strength coupling together the stability and statistical models.

The proposed damage model formulation presented accounts for the fact that during the compressive loading process, as the misaligned fibers become unstable, the load redistributes among the remaining stable fibers. Therefore, failure of highly misaligned fibers does not trigger complete failure unless the compressive load reaches its apex. In most damage model formulations, the damage process is considered as irreversible, resulting in fiber breakage accumulation, etc. In the fiber instability damage model, assuming the postbuckling response to remain elastic, the process is reversible. Hence, when unloaded, the damage variable $\omega(\alpha)$ returns to zero. In the case in which the deformations become plastic or large and fiber kinks form, the reversibility of the system becomes invalid.

Example Correlation with Experimental Compressive Strength

Although the literature contains an abundance of experimental compression strength data for various fiber and matrix systems, relatively few investigations measure all the required information needed for model validation and correlation. For accurate correlation with the model presented in the previous sections, the following information about the composite system is required:

1. Ultimate shear strength, τ_{ULT}.
2. Initial shear modulus, G_{LT}.
3. Fiber misalignment standard deviation, σ_α.
4. Longitudinal compressive strength, σ_{CR}.

Since the complete combination of these parameters is difficult to find in the current literature, a sample correlation will be presented for a graphite/epoxy system (AS4/E7K8). Using data from MIL-HDBK-17-2C [*19*], the longitudinal compressive strength of AS4/E7K8 was found as 209 ksi using a SACMA 1-88 test method (which uses an ASTM Test Method for Compressive Properties of Rigid Plastics [Metric] (D 695) test fixture). When this value is normalized by the fiber volume fraction of 60%, a value of 245 ksi (1.69 GPa) is presented. Unfortunately, shear and misalignment data were not presented with the dataset.

Since shear and misalignment data were not presented with the dataset given in MIL-HDBK-17-2C, separate shear and misalignment measurements on AS4/E7K8 were performed. A typical shear curve for this system is shown in Fig. 1 (two tests) using the Iosipescu test fixture according to ASTM Test Method for Shear Properties of Composite Materials by the V-Notched Beam Method (D 5379) along with the hyperbolic curve-fit (Eq 1), which was used for the stability model formulation. From the curve-fit procedure, a value of the initial shear modulus and ultimate shear strength was found as

$$G_{LT} = 1.15\,\text{Msi}\ (7.93\,\text{GPa})$$

$$\tau_{ULT} = 13.2\,\text{ksi}\ (91.0\,\text{MPa})$$

These values were obtained from a 0° laminate configuration and then subsequentially modified by the correction factor given in Ref *11*.

Using a microscopic video analysis system, measurements of the fiber misalignment were also performed according to the procedure outlined by Yurgartis [*8*]. A typical field of view for an AS4/E7K8 composite is shown in Fig. 6 with approximately a 5° sectioning angle. By measuring both the major and minor axes of the resulting ellipses and correcting for measurement sensitivity by discarding the lower half of the assumed normal distribution [*8*], a standard deviation based on 2016 measurements was obtained for the AS4/E7K8 as

$$\sigma_\alpha = 1.179\,\text{degrees} = 0.02058\,\text{rad}$$

It should also be noted that since a variation of misalignment distributions is expected with respect to the manufacturing process, the laminates manufactured for this analysis were produced using recommended prepreg manufacturer processing specifications.

Using the stability model [*9,10*] along with the obtained shear response values, an imperfection sensitivity curve was established for this system with respect to fiber misalignment (Fig. 5) and was modeled using Eq 3. It should be noted that the use of the model presented in Refs *9* and *10*, although it is crucial to the overall compressive strength prediction, does not depend on the formulation used to obtain the imperfection sensitivity relation. Even though minor differences may result from different formulations, the imperfection sensitivity curves given in Refs *1–4* may also be utilized with the damage mechanics formulation.

Using the standard deviation of the measured misalignment magnitudes, Eq 9 (half normal) was used to model the misalignment distribution. The resulting distribution is then insensitive to the sign of the misalignment and depends only upon the absolute value of the misalignment. In this sense, both positive and negative degrees of misalignment will effect the compressive strength identically.

Following the solution schematic presented in Fig. 11, both models (statistical and stability) are combined together using Eq 10. As previously stated, the use of Eq 10 accounts for the fact that once an unstable fiber has reached its limit load, the load must be

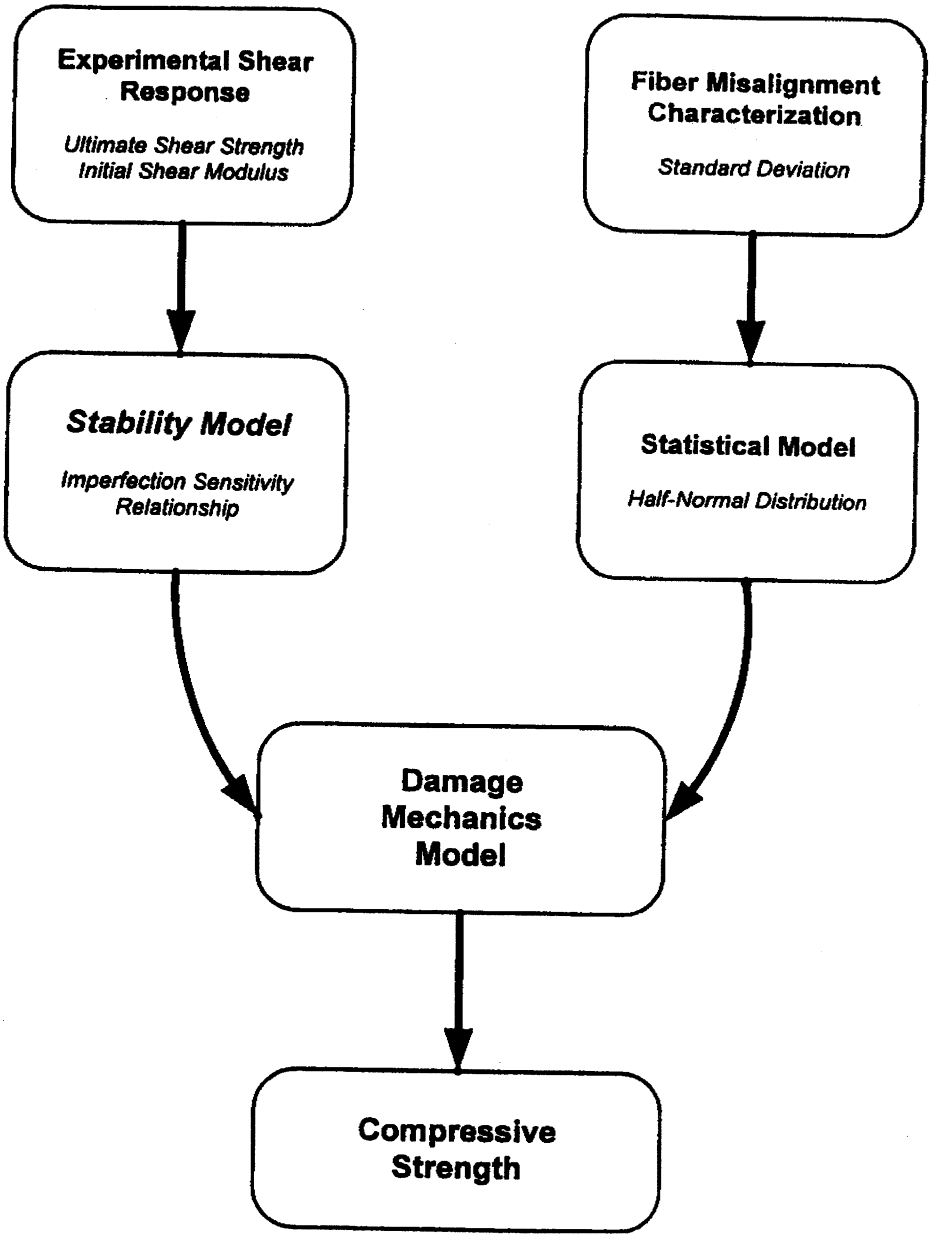


FIG. 11—*Solution scheme for damage mechanics formulation.*

redistributed onto the remaining *stable* fibers. When this reloading process reaches the critical stage, the compressive strength may be found. For the AS4/E7K8 system considered in this example, the maximum compressive stress was found as

$$\sigma_{max} = 235\,\text{ksi}\,(1.62\,\text{GPa})$$

By correlating this result with the experimentally measured strength, a difference of 12 and 4% is obtained with respect to the raw and normalized experimental results, respectively.

Discussion and Conclusions

By using the example presented in the previous section, the correlation of the analytical model with respect to experimentally measured strengths was presented. As seen from this correlation, relatively good predictions may be obtained for the compressive strength without assuming a constant misalignment value for the composite system. By using the imperfection sensitivity curve, a constant misalignment of approximately 2.5° is required to yield the experimentally observed compressive strength. This value (2.5°) does not correlate with either the standard deviation (1.179°), the expected value of the half-normal distribution (0.941°), nor the variance of the half-normal distribution (0.505°) of the measured misalignment distribution. Hence, the model presented provides a *logical* correlation between the experimentally measured fiber misalignment distributions and the experimentally observed compressive strength values.

Although the model presented provides relatively good correlation with the experimentally measured compressive strength, various aspects of the model need further investigation and are listed below:

1. Good correlation of the current model has also been obtained using a glass/polyester system [*10*] in which substantially more variation of the misalignment magnitudes exists. Additional correlations with other material systems are needed for complete model validation.
2. Parametric studies are also required correlating the effects of the initial shear modulus, ultimate shear strength, and fiber misalignment distributions with respect to experimental compressive strength magnitudes.
3. Since the compressive strength is dependent upon the shear response of the composite, accurate methods to measure the nonlinear response are required.
4. Constitutive models of the nonlinear shear response are also required. If a simpler equation may be used to model the nonlinear shear behavior, the use of Eq 3 may be eliminated and a closed form solution of the imperfection sensitivity may be possible.
5. Interaction among the fibers and the effects of nonuniform spacing is not included.
6. The point at which plastic instability/kinks occur with respect to the loading process is unknown.

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