Tsai equation is more suitable for the use on the laminae with greater fiber aspect ratios.

In Figure 5 the predicted transverse modulus E_{TC} and axial shear modulus G_{AC} of the composite (with $\lambda = 20$, $\mu = .05$) are compared with the predictions of the Halpin-Tsai equation. It is seen that with the increasing of fiber volume fraction, the Halpin-Tsai equation may give a higher and higher estimation. Because generally, for E_{TC} and G_{AC}, the predicted values in the present theory are only slightly influenced by the parameters λ and μ, this observation may also be taken as general.

Figure 6 gives the typical features of the stress state in a unit cell of an aligned short-fiber reinforced lamina under axial tension (including tensile stress in fiber and normal and shearing stresses on the bonding surface of the two phases). They are qualitatively in agreement with the earlier expectations in the literature [1], but are more elaborate and specific.

In Figure 7 the two thermal expansion coefficients α_{AC}, α_{TC} for a typical lamina ($\lambda = 20$, $\mu = 0.05$) are given against the fiber volume fraction, and are compared with those for the corresponding continuous-fiber reinforced composite [2,8]. It is seen that in this case the differences in thermal expansion coefficients caused by fiber aspect ratio are not significant.

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Experimental Evaluation of Stiffness of Laminated Composite Beam Elements Under Flexure

ROBERTO LOPEZ-ANIDO*, JULIO F. DAVALOS * AND EVER J. BARBERO**
West Virginia University
Constructed Facilities Center
Morgantown, WV 26506-6101

ABSTRACT: The objectives of this study are: (1) to define upper and lower bounds for the stiffness of laminated beam elements under flexure, (2) to correlate experimental effective beam moduli with analytical predictions, and (3) to obtain a threshold aspect-ratio that limits the range of application of various analytic methods. Experimental tests were conducted to characterize the load-displacement response of laminated beam elements under flexure, and the results were correlated with available analytic approaches. Angle-ply laminated beam elements were chosen to distinctly show the differences between various analytic methods. Three-point bending tests were performed, both flatwise (out-of-plane) and edgewise (in-plane), on rectangular lay-up specimens made of AS-4/3501-6 carbon-epoxy. Recommendations are made for the analytical prediction of stiffness under various conditions.

KEY WORDS: composite materials, beam theory, bending tests, angle ply composites, laminated beams, beam stiffness.

INTRODUCTION

LAMINATED COMPOSITE BEAM elements constitute the basic components of thin-walled beams, or may represent self-standing thin rectangular beams. Furthermore, they can be used as reinforcement or encasing of beams and columns made of concrete or glued-laminated timber [1]. Three different methodologies are available in the literature to evaluate the stiffness of laminated composite beam elements under flexure.

First, Bert [2] proposed beam stiffnesses for rectangular laminated beams based on an integration through the thickness of the piecewise-constant lamina longitudinal and shear moduli. Vinson and Sierakowski [3] applied this approach to develop "an advanced theory," as they call it.

*Department of Civil Engineering.
**Author to whom correspondence should be addressed; Department of Mechanical and Aerospace Engineering.

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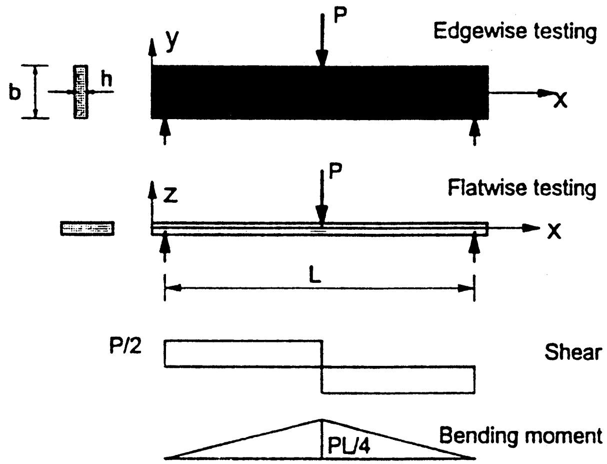

Figure 1. *Three-point bending tests: geometry, loading and stress resultants.*

Second, stiffness coefficients for laminated beams were proposed by Vinson and Sierakowski [3] by extending classical laminated plate theory to a simple theory for beams. They derived beam stiffness coefficients from the corresponding laminate stiffness coefficients by neglecting lateral strains and curvatures. Bauld and Tzeng [4] presented a Vlasov-type theory for thin-walled laminated beams with open cross-section based on this criterion.

Third, effective moduli of laminated beams were proposed by Whitney et al. [5] and Tsai [6] from the reciprocals of the corresponding laminate compliances. Barbero et al. [7] adopted this approach in order to develop a first-order shear deformation theory for thin-walled laminated beams; they modeled a thin-walled beam as an assembly of laminated panels with stiffnesses characterized by effective beam moduli.

For thin-walled beams, other authors (see, for example, Wu and Sun [8], and Skudra et al. [9]) applied a combination of the second and third approaches presented herein to compute laminated beam stiffnesses depending on the element slenderness and the presence of stiffening ribs. While for orthotropic laminated beams the three approaches mentioned earlier yield almost coincident results, for general laminated beams that incorporate off-axis layers, they predict different stiffnesses.

Whitney et al. [5] presented an experimental analysis of laminated beams. They tested flatwise angle-ply and quasi-isotropic graphite/epoxy laminated specimens with various span-to-width ratios, and they showed the importance of beam dimensions on mechanical effects. Edgewise tests of laminated beam specimens are rare in the literature due to difficulties in the test setup. In this work, in order to characterize the load-displacement response of laminated beam elements under flatwise and edgewise flexure, three-point bending tests were conducted, and the results were correlated with available analytic methods. Angle-ply ($\pm 45s$) laminated beam elements were chosen to distinctly show the differences among various analytic approaches. The load-deflection ratio (P/δ) is studied as a function of the length-to-width aspect ratio (L/b) (see Figure 1).

The objectives of this work are: (1) to define upper and lower bounds for the stiffness of laminated beam elements under flexure, (2) to correlate the experimental in-plane (edgewise) and out-of-plane (flatwise) beam effective moduli with analytical predictions, and (3) to obtain a threshold aspect-ratio that limits the range of application of the analytic methods.

MATERIALS AND MANUFACTURING

Hercules AS4/3501-6 carbon prepreg tape, which is an amine-cured epoxy resin with unidirectional carbon fibers, was used to manufacture the composite. The prepreg plies were cut at 45 degrees, and then, the laminate was manufactured with 24 plies, which resulted in a total final thickness of approximately 2.85 mm (0.112 in). The composite was made by hand lay-up in a sequence of $[\pm 45]_{12s}$. A vacuum bag was assembled prior to the curing process. The thermoset curing was performed by increasing the temperature in steps up to 177°C (350°F) while maintaining the pressure at 0.59 MPa (85 psi). A PHI-Hydraulic Compression Press, which has full control over temperature, time, and event (cooling, heating and force), was utilized following the program implemented in Reference [10]. A post-cure cycle, that increases the cross-link density, was induced by raising the temperature in steps up to 188°C (370°F). Finally, the specimens were cut in lengths of 267 mm (10.5 in.) with three different widths of approximately 12.7 mm (1/2 in), 19.1 mm (3/4 in), and 25.4 mm (1 in). The exact dimensions of the specimens are shown in Table 1.

The following elastic properties of AS4/3501-6 carbon-epoxy were experimentally determined [10].

$$E_1 = 138 \text{ GPa } (20 \times 10^6 \text{ psi})$$

$$E_2 = 9.65 \text{ GPa } (1.4 \times 10^6 \text{ psi}) \tag{1}$$

$$G_{12} = 5.24 \text{ GPa } (0.76 \times 10^6 \text{ psi}), \text{ and } \nu_{12} = 0.3$$

EXPERIMENTAL PROCEDURE AND DATA

A three-point bending fixture was attached directly to the load cell of an INSTRON Universal Testing System (UTS). A calibrated load cell with a capacity

Table 1. Geometric dimensions of the specimens.

Specimen	IA	IB	IIA	IIB	III
b[mm]	12.78	13.28	20.09	20.04	47.42
h[mm]	2.92	2.87	2.85	2.95	2.95
b/h	4.37	4.63	7.06	6.80	16.09

Table 2. Length-to-width aspect ratios of the beam elements.

	L/b				
L[mm]	IA	IB	IIA	IIB	III
127.79	10.00	9.62	6.36	6.38	2.69
181.76	14.23	13.68	9.05	9.07	3.83
233.38	18.27	17.57	11.62	11.65	4.92

of 445 N (100 lbs.), and a precision of 0.1 mN (0.02 lbs.) was used. A Linear Voltage Differential Transducer (LVDT) with 7.6 mm (0.3 in) range was used to measure midspan deflections. A flat grip with a 12.7 mm (1/2 in) hole for mounting the LVDT was attached to the loading rod. The LVDT readings were made directly between the loading rod and the fixture base, that is the readings corresponded exactly to the maximum deflection in the sample. The Data Acquisition System consisted of a 386 PC, a 12 bit A/D Metrabyte data acquisition card, and Labtech Notebook software [11]. Five data points were recorded per second. An aluminum-alloy sample, 2014-T6, with elastic modulus $E = 73.1$ GPa (10.6×10^6 psi), was tested to validate the experimental load-deflection prediction. For the aluminum sample, a difference of less than 2% was obtained between the experimental deflections and the analytical predictions.

The three-point flexure test was set up as described by Carlsson and Pipes [12] for a flatwise geometry. The beam elements were loaded in the x-z plane for the flatwise position, and in the x-y plane for the edgewise position, as shown in

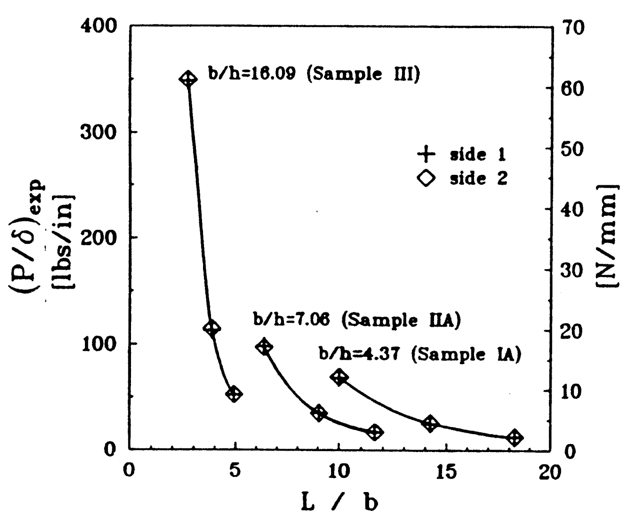

Figure 2. Load-deflection coefficients versus aspect ratio for flatwise testing.

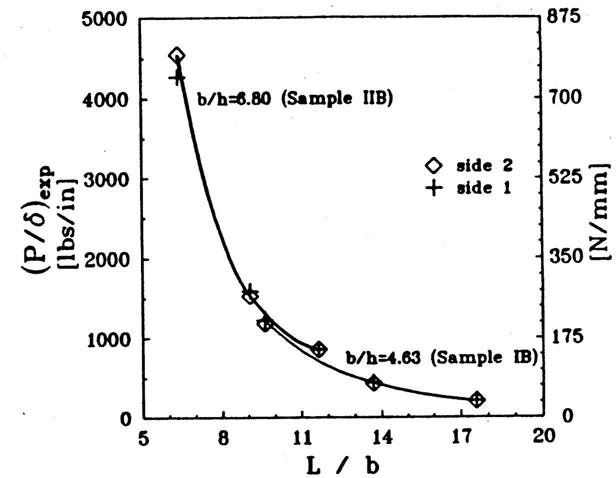

Figure 3. Load-deflection coefficients versus aspect ratio for edgewise testing.

Figure 1. The specimen lengths and length-to-width aspect ratios are given in Table 2. A general computer program [13] was developed to compute the laminate stiffness and compliance coefficients, as well as ply strains and stresses, and to evaluate various failure criteria. In order to load the specimens within the linear elastic range, a safety factor of 1.5 was applied to the predicted load causing first-ply failure by the Tsai-Wu criterion [14]. In the flatwise position, 18 tests were performed with a loading rate of 1.3 mm/min. (0.05 in/min.), while in the edgewise position, 12 tests were performed at a lower loading rate of 0.5 mm/min. (0.02 in/min.), to obtain sufficient number or data points.

An initial slack in the load-displacement curves was more pronounced for the edgewise testing due to initial instability of the specimen. Therefore, a preload was used in the calculation of the load-displacement ratio by linear regression on the experimental data. The load-deflection coefficients versus the length-to-width aspect ratio are depicted in Figure 2 for the specimens tested flatwise, and in Figure 3 for the specimens tested edgewise. The minimum aspect ratio tested in the edgewise position was $L/b = 6.4$ due to stability constraints. In both Figures 2 and 3, we observe different P/δ curves for different values of the ratio b/h.

ANALYTICAL PREDICTION OF EFFECTIVE BEAM MODULI

Three different methodologies have been used in the literature for the evaluation of the stiffness coefficients (effective beam moduli) of beam elements.

The first approach, proposed by Bert [2] for rectangular laminated beams, is based on integrating through the thickness the piecewise-constant lamina longitudinal and shear moduli. Using the same criterion, Vinson and Sierakowski [3] presented an " advanced theory" for bending of laminated beams. The apparent lamina moduli used in this approach are obtained from the reciprocals of the corresponding transformed lamina compliances ($\bar{S}_{ij}$) (see, for example, Jones [14]). The apparent lamina moduli are generally used to correlate the results of tests performed on a single off-axis lamina; e.g., uniaxial tensile loading or pure shear loading. Modeling the stiffness of a laminated beam by adding the contribution of the lamina moduli through the thickness implies that the layers are not constrained and they do not interact with one another. In other words, the laminated beam is viewed as a one-dimensional arrangement of parallel springs [16]. Hence, such an approach is defined herein as an *arrangement of laminae*. For pultruded structural shapes, Barbero [15] proposed a constitutive relation on each layer based on extensional (E_x) and shear (G_{xz}) moduli that were derived by assuming that only the stress components σ_x and σ_{xz} are different from zero. The moduli E_x and G_{xz}, that were expressed in terms of lamina stiffness components, resulted identical to the corresponding reciprocals of the lamina compliance components ($\bar{S}_{ij}$). This criterion was adopted by Davalos et al. [17] in order to develop a 2-D laminated beam finite element with layer-wise constant shear. The transformed lamina compliance coefficients related to extension in the beam length direction and in-plane shear are $\bar{S}_{11}$ and $\bar{S}_{66}$, respectively. In-plane extensional and shear stiffness coefficients, as well as out-of-plane flexural stiffness coefficients may be computed following this methodology as

$$\tilde{A} = \int_{-h/2}^{h/2} \frac{1}{\bar{S}_{11}}\, dz$$

$$\tilde{F} = \int_{-h/2}^{h/2} \frac{1}{\bar{S}_{66}}\, dz \tag{2}$$

$$\tilde{D} = \int_{-h/2}^{h/2} \frac{1}{\bar{S}_{11}}\, z^2 dz$$

A second approach proposed by Vinson and Sierakowski [3] as a "simple theory " for beams, obtains beam stiffness coefficients from the laminated plate stiffness coefficients by assuming that lateral strains and curvatures are zero:

$$\epsilon_y = \varkappa_y = 0 \tag{3}$$

This criterion is consistent with plane strain assumptions employed, for example, in cylindrical bending of plates, i.e., the beam element is viewed as a plate strip under a state of plane strain. Therefore, this approach is referred to herein as *plane strain,* and the following normalized membrane and flexural stiffness coefficients are obtained for a laminated beam:

$$A_{11}^{*} = \frac{A_{11}}{h}$$

$$A_{66}^{*} = \frac{A_{66}}{h} \tag{4}$$

$$D_{11}^{*} = \frac{12D_{11}}{h^3}$$

where A_{11} and A_{66} are components of the laminate in-plane stiffness matrix, and D_{11} is a component of the laminate flexural stiffness matrix. Following the same approach, Bauld and Tzeng [4] developed a Vlasov-type theory for thin-walled laminated beams with open cross-section. For each plate element of the cross-section, they introduced analogous normalized stiffness coefficients (A_{ij}/h, D_{ij}/h^3).

The third approach, proposed by Whitney et al. [5] and Tsai [6], considers that the effective moduli of a laminated beam are the corresponding engineering constants of the laminate. Hence, the effective beam moduli are obtained from the reciprocals of the components of the laminate compliance matrix, which is obtained by full inversion of the laminate stiffness matrix. The basic assumption in this approach is that the lateral force and moment stress resultants in a beam element are zero, as follows

$$N_y = M_y = 0 \tag{5}$$

This approach is consistent with plane stress assumptions employed in conventional beam theory, and therefore, is termed *plane stress-resultant*. Thus, the membrane (in-plane) beam stiffness is characterized by the following effective moduli

$$E_x^{\circ} = \frac{1}{\alpha_{11}h}$$

$$G_{xy}^{\circ} = \frac{1}{\alpha_{66}h} \tag{6}$$

where α_{11} and α_{66} are components of the laminate in-plane compliance submatrix [6]. In a similar way, the flexural (out-of-plane) beam stiffness follows from the corresponding effective modulus:

$$E_x^{f} = \frac{12}{\delta_{11}h^3} \tag{7}$$

where δ_{11} is a component of the laminate flexural compliance submatrix [6]. Based on this approach, Barbero et al. [7] presented a first-order shear deformation theory for thin-walled laminated beams. They computed the stiffnesses of a thin-walled beam by adding the contribution of the stiffnesses of the component panels, which in turn were obtained from the effective beam moduli.

Wu and Sun [8] analyzed laminated composite thin-walled beams by finite elements. The wall stiffnesses were computed by assuming a rule of mixtures between the stiffnesses predicted by the second and the third approaches presented herein. They suggested that for cross-sections stiffened by closely spaced ribs, the plane strain approach should yield more accurate results. Conversely, they stated that for slender beams without ribs, the plane stress-resultant approach should yield more accurate results. Skudra et al. [9], proposed similar recommendations for thin-walled laminated beams. They recommended to use the plane strain assumption when stiff diaphragms are attached along the beam length or when the beam acts in combination with sheathing or light filling. Otherwise, they considered that plane stress-resultant assumptions are correct.

While for orthotropic laminated beams the three approaches produce almost identical beam moduli, for general laminated beams that incorporate off-axis layers, these theories predict different results, as follows

$$\begin{aligned} \frac{\tilde{A}}{h} &\le E_x^{\circ} < A_{11}^{*} \\ \frac{\tilde{F}}{h} &\le G_{xy}^{\circ} = A_{66}^{*} \\ \frac{12\tilde{D}}{h^3} &\le E_x^{f} < D_{11}^{*} \end{aligned} \tag{8}$$

For an angle-ply laminated beam ($\pm\phi$) with $0° < \phi < 90°$, Equation (8) reduces to the following relations

$$\begin{aligned} \frac{1}{\bar{S}_{11}} &< E_x^{\circ} < \bar{Q}_{11} \\ \frac{1}{\bar{S}_{66}} &< G_{xy}^{\circ} = \bar{Q}_{66} \\ \frac{1}{\bar{S}_{11}} &< E_x^{f} < \bar{Q}_{11} \end{aligned} \tag{9}$$

where $\bar{Q}_{11}$ and $\bar{Q}_{66}$ are components of the lamina transformed reduced stiffness matrix [14]. Using the three different approaches described in this section, the beam moduli for the angle-ply test specimens used in this study are computed as shown in Tables 3 and 4 for flatwise and edgewise testing, respectively.

Table 3 Effective beam modulus for out-of-plane testing (flatwise).

Approach	$(E_x^f)_{anal}$ [GPa]	
Arrangement of laminae	$12\tilde{D}/h^3$	13.44
Plane strain	$12\,D_{11}/h^3$	43.79
Plane stress-resultant	$12/(\delta_{11}h^3)$	18.42

EXPERIMENTAL-ANALYTICAL CORRELATIONS

Following the mechanics of thin laminated beam [7], which has the same form as Timoshenko beam theory, we consider shear deformations for the in-plane loaded specimens (edgewise), where as, we neglect shear deformations for the out-of-plane loaded specimens (flatwise). Therefore, the midspan deflection for flatwise testing results in

$$\delta = \frac{PL^3}{48\,(E_x^f)_{anal}I_y} \tag{10}$$

where, $I_y = bh^3/12$, and $(E_x^f)_{anal}$ is the effective beam bending modulus. For edgewise testing, the midspand deflection is

$$\delta = \frac{PL^3}{48\,(E_x^{\circ})_{anal}I_z} + \frac{PL}{4K\,(G_{xy}^{\circ})_{anal}A} \tag{11}$$

where, $I_z = hb^3/12$, $A = bh$, and $(E_x^{\circ})_{anal}$ and $(G_{xy}^{\circ})_{anal}$ are the effective beam in-plane extensional and shear moduli. The effective beam moduli in Equations (10) and (11) are enclosed in parentheses to emphasize that they can be computed from either of the analytic approaches presented previously. The shear stiffness in Equation (11) is corrected by a shear coefficient $K = 5/6$, which is based on the derivations presented in [7].

For experimental-analytical correlations, it is convenient to represent the different experimental P/δ curves for flatwise bending (Figure 2) by a single curve, which can be obtained by rearranging Equation (10) as

Table 4. Effective beam moduli for in-plane testing (edgewise).

Approach	$(E_x^{\circ})_{anal}$ [GPa]		$(G_{xy}^{\circ})_{anal}$ [GPa]	
Arrangement of laminae	$\tilde{A}/h$	13.44	$\tilde{F}/h$	8.67
Plane strain	A_{11}/h	43.79	A_{66}/h	35.64
Plane stress-resultant	$1/(\alpha_{11}h)$	18.44	$1/(\alpha_{66}h)$	35.64

$$\left(\frac{P}{\delta}\right)_{exp}\frac{L^2}{h^3} = 4\,\frac{(E_x^f)_{anal}}{\dfrac{L}{b}} \tag{12}$$

where $(P/\delta)_{exp}$ is experimentally measured, and the effective beam modulus $(E_x^f)_{anal}$ is computed by the analytic approaches presented in Table 3. Similarly, the experimental P/δ curves for edgewise bending (Figure 3) can be represented by a single curve by rearranging Equation (11) as

$$\left(\frac{P}{\delta}\right)_{exp}\frac{L^2}{b^2h} = \frac{4}{\dfrac{\frac{L}{b}}{(E_x^\circ)_{anal}} + \dfrac{\frac{b}{L}}{K(G_{xy}^\circ)_{anal}}} \tag{13}$$

where the effective beam moduli $(E_\circ^x)_{anal}$ and $(G_{xy}^\circ)_{anal}$ are computed by the different analytic approaches given in Table 4. By plotting the left-hand sides of Equations (12) and (13) using only the experimental data, the experimental P/δ curves of Figures 2 and 3 are each reduced to single curves as shown in Figures 4 and 5. In the same manner, by plotting the right-hand side of Equations (12) and (13) using only analytical predictions, continuous curves are obtain in Figures 4 and 5 for the response of the flatwise and edgewise samples, respectively. In both, Figures 4 and 5, we note that the approach termed arrangement of laminae yields a lower bound, while the approach based on assuming a *plane strain* state

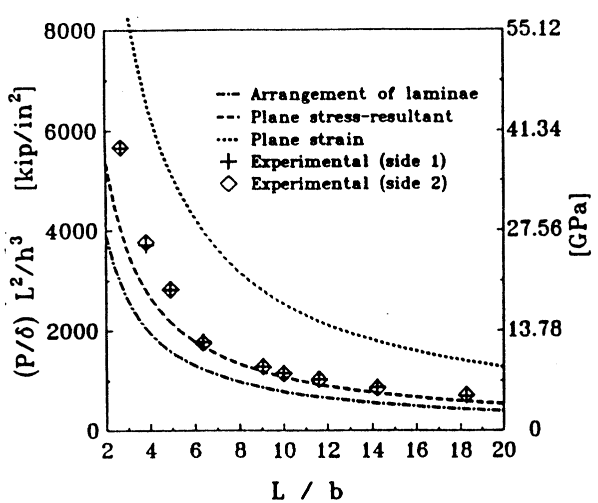

***Figure 4.** Correlation of experimental stiffness with analytical predictions for flat-wise testing.*

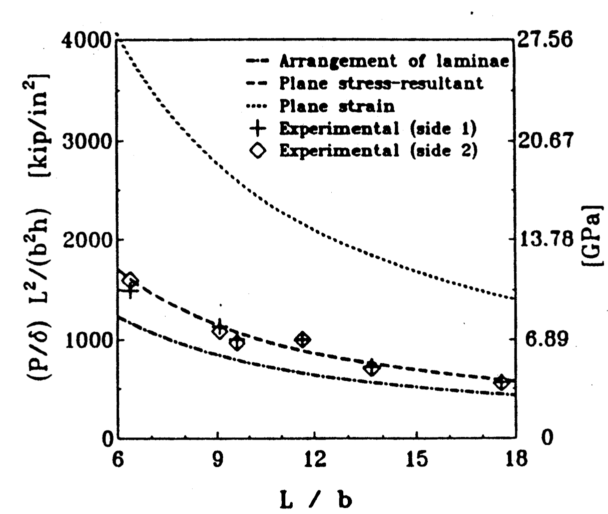

***Figure 5.** Correlation of experimental stiffness with analytical predictions for edge-wise testing.*

provides an upper bound for the beam stiffnesses. The results show that, the experimental data support Equations (8) and (9), and that the effective beam moduli obtained from *plane stress-resultant* assumptions correlate very well with the experimental beam stiffness for aspect ratios

$$\frac{L}{b} \geq 6 \tag{14}$$

Equation (14) provides a threshold to characterize beam behavior by the *plane stress-resultant* assumptions. In Figure 4, for aspect ratios $L/b < 6$ the experimental stiffness departs from the plane stress-resultant curve and approaches the plane strain curve. For very long beams, $L/b \gg 20$, both *arrangement of laminae and plane stress-resultant* approaches yield very closed results, i.e., the effects due to constraints and interactions between layers become negligible with the increase in the aspect ratio L/b. The experimental stiffness response shown in Figure 4 for aspect ratios $L/b < 6$, supports the analysis of thin-walled beams stiffened by ribs as discussed in References [8] and [9], provided the length (L) is interpreted as the rib spacing.

CONCLUSION AND DISCUSSION

Three-point bending tests were performed on lay-up angle ply ($\pm 45s$.) beam elements made of AS-4/3501-6 carbon-epoxy. The load-deflection response as a function of the length-to-width aspect ratio (L/b) was investigated. For the samples selected for this study, the three analytic approaches discussed in this work predicted different responses. Two types of configurations were tested: flatwise and edgewise. The main findings of the present study are:

- The analysis based on the computation of the apparent lamina moduli (*arrangement of laminae*) provides a lower bound for the beam stiffness.
- The analysis based on *plane strain* assumptions represents an upper bound for the beam stiffness.
- The effective beam moduli obtained from *plane stress-resultant* assumptions as laminate engineering constants, represent the actual beam stiffness for aspect ratios $L/b \geq 6$.
- For aspect ratios $L/b < 6$, the actual stiffness is represented by a combination of the *plane stress-resultant* and the *plane strain* approaches.
- The present conclusions may be extended to beam elements stiffened by ribs or diaphragms, by interpreting the length (L) as the rib spacing. Thus, this work provides a criterion to relate the laminated beam element stiffness to the rib spacing.

In order to generalize the conclusions presented herein to thin-walled laminated beams made of assemblies of beam elements, consideration should be given to the effect of warping of the cross-section due to shear deformation.

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