

# Lateral and distortional buckling of pultruded I-beams

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The elastic buckling modes of pultruded I-beams subjected to various loading conditions are studied. The coupling of lateral and distortional buckling, very likely to appear in thin-walled cross-sections, is investigated. Plate theory is used to allow for distortion of the cross-section. Shear effects and bending-twisting coupling are accounted for in the analysis, because of their significant role. The effect of fiber orientation in the matrix and volume fraction is investigated by a parametric study. Pultruded cross-sections are always thin-walled due to constraints in the manufacturing process. Hence, the buckling strength determines the overall strength of the member. The results presented correspond to actual pultruded cross-sections being used in civil engineering types of structure.

### INTRODUCTION

Various pultrusion manufacturers produce on an industrial basis beams with a variety of crosssectional shapes and dimensions (e.g. I-beams, wide flange I-beams, box-beams, angle beams, tubes, etc.). These products are made from polymers (usually called resin in the uncured state and matrix in the cured state) with fiber reinforcement. Polyester, vinylester or epoxy are used as a matrix to hold together E-glass, S-glass, Kevlar or carbon fibers used as reinforcement. Fibers and polymer are joined through the pultrusion process to form the desired cross-section. The present study is concerned with the stability of I-beams and wideflange I-beams under bending loads.

An I-beam can buckle with various modes depending on the geometry of the cross-section, the material properties, and the boundary and loading conditions. The beam can buckle either locally, or laterally, or with a combination of local and lateral modes. Local buckling is defined as the instability mode when changes in the geometry of the cross-section occur, but not accompanied by lateral displacement or twist. Each part of the cross-section (flanges and web) may buckle as a plate. The coupling of the local buckling of flanges and web may also occur. In the case of lateral buckling, there is a lateral displacement and twist of the cross-section without local changes in the cross-section geometry.

The most general case is when coupling between local modes and lateral modes of buckling occurs. This is called distortional buckling and in many cases of beams with certain dimensions, distortional buckling can result in a significant lowering of the critical load.<sup>1-3</sup>

Many studies have been done separately on the local buckling<sup>4-9</sup> and lateral buckling of I-beams, but only a few on a combination of the buckling modes (distortional buckling).<sup>3</sup> The material used in previous studies on lateral and distortional buckling is steel (homogeneous and isotropic) with the exception of Mottram<sup>10,11</sup> who considered lateral buckling of an orthotropic material. Pultruded cross-sections are thin-walled and each part of the cross-section is treated in this work as a laminated plate. Each lamina can be either specially orthotropic or generally orthotropic. The stiffness coupling terms are important, especially when bending-twisting coupling terms are present because they produce higher instabil-

ity. The results presented correspond to existing pultruted cross-sections.

### **THEORETICAL APPROACH**

The energy criterion for equilibrium of a structure is that the first variation of the total potential energy is zero

$$\delta V = 0 \tag{1}$$

The state of equilibrium when the system loses its stability is characterized by the vanishing of the second variation of the total potential energy<sup>12</sup>

$$\delta^2 V = 0 \tag{2}$$

As a first step, we formulate the total potential energy of the system

$$V = \frac{1}{2} \int_{V} \sigma_{ij} \varepsilon_{ij} \, \mathrm{d}V - \sum_{k} P_{k} q_{k}$$
(3)

where  $P_k$  are the externally applied forces and  $q_k$  are the corresponding displacements.

Considering first the energy terms corresponding to the web for the coordinate system shown in Fig. 1, we use the von-Karman non-linear strains in terms of displacements to describe the kinematics of the system

$$\varepsilon_{x} = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^{2}$$

$$\varepsilon_{y} = \frac{\partial v}{\partial y} + \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^{2}$$
(4)

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + 2 \frac{\partial w}{\partial x} \frac{\partial w}{\partial y}$$

The constitutive law for the web considered as a layered plate (laminate) is given from the Clas-



Fig. 1. Coordinate system and geometry of the crosssection. (a) Coordinate system, (b) cross-section, (c) flange.

sical Lamination Theory<sup>13</sup> as

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$
(5)

where  $Q_{ij}$  are 'rotated' stiffness quantities corresponding to the global coordinate system (Fig. 1).

Substituting eqns (4) and (5) into eqn (3) we get an expression for the total potential energy for the web in terms of displacements. The second variation of the total potential energy for the web is obtained by performing the following substitutions (eqn (6) into eqn (3))

$$u \to u + \delta u$$
  

$$v \to v + \delta v$$
  

$$w \to w + \delta w$$
  
(6)

and collecting the second order terms. The membrane forces  $N_x$ ,  $N_y$  and  $N_{xy}$  can be expressed in terms of strains as follows

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$
(7)

Similarly, the bending and twisting moments  $M_x$ ,  $M_y$  and  $M_{xy}$  can be expressed as

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$
(8)

where  $A_{ij}$  and  $D_{ij}$  are the extensional stiffness matrix and the bending stiffness matrix respectively. These matrices can be derived using the Classical Lamination Theory<sup>13</sup> for a layered plate. For an *N*-layered laminated plate the stiffness terms are given by

$$A_{ij} = \sum_{k=1}^{N} (Q_{ij})_k t_k$$

$$D_{ij} = \sum_{k=1}^{N} (Q_{ij})_k \left( z_k^2 t_k + \frac{t_k^3}{12} \right)$$
(9)

Performing the integration over the thickness and identifying the terms corresponding to the extensional stiffness  $A_{ij}$  and bending stiffness  $D_{ij}$ , we obtain the following expression for the second variation of the total potential energy for the web

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10)

$$\delta^{2} V_{w} = \frac{1}{2} \iiint \left[ N_{x}^{w} \left( \frac{\partial \delta w}{\partial x} \right)^{2} + N_{y}^{w} \left( \frac{\partial \delta w}{\partial y} \right)^{2} + 2 N_{xy}^{w} \frac{\partial \delta w}{\partial x} \frac{\partial \delta w}{\partial y} \right] dx dy \\ + \frac{1}{2} \iiint \left[ D_{11}^{w} \left( \frac{\partial^{2} \delta w}{\partial x^{2}} \right)^{2} + D_{22}^{w} \left( \frac{\partial^{2} \delta w}{\partial y^{2}} \right)^{2} + 2 D_{12}^{w} \frac{\partial^{2} \delta w}{\partial x^{2}} \frac{\partial^{2} \delta w}{\partial y^{2}} + 4 D_{66}^{w} \left( \frac{\partial^{2} \delta w}{\partial x \partial y} \right)^{2} \\ + 4 D_{16}^{w} \frac{\partial^{2} \delta w}{\partial x^{2}} \frac{\partial^{2} \delta w}{\partial x \partial y} \\ + 4 D_{26}^{w} \frac{\partial^{2} \delta w}{\partial y^{2}} \frac{\partial^{2} \delta w}{\partial x \partial y} \right] dx dy \qquad (4)$$

The term  $\Sigma P_k \delta^2 q_k$  does not appear in the above expression, because  $q_k$  can be expressed as linear functions and hence  $\delta^2 q_k$  vanishes. The terms involving the extensional stiffnesses  $A_{ij}$  have been omitted from the above expression because they are related to the second variation of the displacements that are always positive definite.<sup>3</sup>

Considering now the flanges, they can bend and twist as plates and also bend laterally as beams. When the flange bends as a beam, the total potential energy is

$$\boldsymbol{V}^{\mathbf{p}} = \int \boldsymbol{\sigma}_{x} \boldsymbol{\varepsilon}_{x} \, \mathrm{d} \, \boldsymbol{V} - \sum_{k} \boldsymbol{P}_{k} \boldsymbol{q}_{k} \tag{11}$$

The non-linear axial strain  $\varepsilon_x$  in terms of displacement is

$$\boldsymbol{\varepsilon}_{x} = \frac{\mathrm{d}\boldsymbol{u}_{\mathrm{f}}}{\mathrm{d}\boldsymbol{x}} + \frac{1}{2} \left(\frac{\mathrm{d}\boldsymbol{w}_{\mathrm{f}}}{\mathrm{d}\boldsymbol{x}}\right)^{2} \tag{12}$$

For a flange in pure bending about the strong axis of the flange (Fig. 1)

$$N_{z} = N_{xz} = M_{z} = M_{xz} = 0$$
  

$$N_{x} = A\varepsilon_{x}; \quad M_{x} = D\kappa_{x}$$
(13)

For a laminate without bending-extension coupling, the beam extensional and bending coefficients are<sup>16</sup>

$$A = \frac{1}{\alpha_{11}}; \quad D = \frac{1}{\delta_{11}}$$
(14)

where the compliance coefficients are obtained by inversion of the stiffness matrices eqn(7) and(8)

$$[\alpha] = [A]^{-1}; [\delta] = [D]^{-1}$$
(15)

Substituting the expressions (12) and (14) into eqn (11) we obtain the potential energy in the flange due to lateral bending. Performing the following substitutions,

$$u_{f} \rightarrow u_{f} + \delta u_{f}$$

$$w_{f} \rightarrow w_{f} + \delta w_{f}$$
(16)

and collecting second order terms we obtain an expression for the second variation of the total potential energy for the flanges bending as beams,

$$\delta^{2} V_{f}^{b} = \frac{1}{2} b_{f} \int N_{x}^{f} \left( \frac{\partial \delta w_{f}}{\partial x} \right)_{T,B}^{2} dx$$
$$+ \frac{h^{3}}{24} A \int \left( \frac{\partial^{2} \delta w_{f}}{\partial x^{2}} \right)_{T,B}^{2} dx \qquad (17)$$

where A is defined in eqn (14). T and B in eqn (17) indicate top and bottom flange energy terms respectively. The extensional stiffness terms omitted in eqn (17) because they are positive definite.

When the flanges bend and twist as plates, the expression for the second variation of the total potential energy is similar to the one for the web,

$$\delta^{2} \mathcal{V}_{f}^{p} = \frac{1}{2} \int \int N_{x}^{t} \left( \frac{\partial \delta v_{f}}{\partial x} \right)_{T,B}^{2} dx dz$$

$$+ \frac{1}{2} \int \int \left[ D_{11}^{t} \left( \frac{\partial^{2} \delta v_{f}}{\partial x^{2}} \right)^{2} + D_{22}^{t} \left( \frac{\partial^{2} \delta v_{f}}{\partial z^{2}} \right)^{2} \right]$$

$$+ 2D_{12}^{t} \frac{\partial^{2} \delta v_{f}}{\partial x^{2}} \frac{\partial^{2} \delta v_{f}}{\partial z^{2}} + 4D_{66}^{t} \left( \frac{\partial^{2} \delta v_{f}}{\partial x \partial z} \right)^{2}$$

$$+ 4D_{16}^{t} \frac{\partial^{2} \delta v_{f}}{\partial x^{2}} \frac{\partial^{2} \delta v_{f}}{\partial x \partial y}$$

$$+ 4D_{26}^{t} \frac{\partial^{2} \delta v_{f}}{\partial y^{2}} \frac{\partial^{2} \delta v_{f}}{\partial x \partial y} \Big|_{T,B} dx dz \qquad (18)$$

where  $N_{y}^{t}$  and  $N_{xy}^{t}$  are zero because only axial forces act on the flange.

For the flanges we assume no distortion (i.e. the displacements are assumed to be linear in z). hence

$$v_{\rm f} = -z\theta_{\rm f} \tag{19}$$
$$\frac{\partial^2 \delta v_{\rm f}}{\partial z^2} = 0$$

and the compatibility equations at the flange-web connections are

$$(w_{\rm f})_{\rm T} = w(x,0) \qquad (w_{\rm f})_{\rm B} = w(x,b_{\rm w})$$

$$(\theta_{\rm f})_{\rm T} = \frac{\partial w}{\partial y}(x,0) \qquad (\theta_{\rm f})_{\rm B} = \frac{\partial w}{\partial y}(x,b_{\rm w})$$
(20)

where  $\theta_{f}$  is the angle of rotation of the flange. Introducing eqn (19) into eqn (18) we get

$$\delta^{2} V_{f}^{p} = \frac{1}{2} \frac{b_{f}^{3}}{12} D_{11}^{f} \int \left( \frac{d^{2}}{dx^{2}} \frac{\partial w}{\partial y} \right)_{T,B}^{2} dx$$
$$+ \frac{1}{2} 4 b_{f} D_{66}^{f} \int \left( \frac{d}{dx} \frac{\partial w}{\partial y} \right)_{T,B}^{2} dx$$
$$+ \frac{1}{2} \frac{b_{f}^{3}}{12} \int N_{x}^{f} \left( \frac{d}{dx} \frac{\partial w}{\partial y} \right)_{T,B}^{2} dx \qquad (21)$$

Since the flange cannot distort, there is no bending-twisting coupling present in eqn (21). Finally, the complete expression for the second variation of the total potential energy for the beam is

$$\delta^2 V = \delta^2 V_{\rm w} + \delta^2 (V_{\rm f}^{\rm b})_{\rm T,B} + \delta^2 (V_{\rm f}^{\rm p})_{\rm T,B}$$
(22)

and the critical condition (buckling instability) is given by eqn (2).

### NUMERICAL RESULTS AND DISCUSSION

The solution of the problem is attempted by the Rayleigh-Ritz method. We select a displacement function for the web as follows

$$\delta w = \sum_{m=1}^{p} \sum_{n=1}^{q} \alpha_{mn} \sin\left(\frac{m\pi x}{a}\right) \left(\frac{y}{b}\right)^{n}$$
(23)

This function can represent a simply supported beam, the web of which can distort when buckling laterally. Introducing eqn (23) into eqn (2) we produce an eigenvalue problem of order  $p \times q$ . Hence the critical buckling load can be computed as a function of the prebuckling solution for the stress resultants  $N_x^w$ ,  $N_y^w$  and  $N_{xy}^w$  in the web and  $N_x^t$  in the flanges.

A variety of lateral-distortional buckling cases for pultruted structural shapes are next investigated. More specifically, five types of different geometry I-beams have been considered, all sim-

ply supported and free to warp at both ends and subjected to two loading conditions: (1) a transverse load applied at the centroid, and (2) bending moments at both ends of the beam (Fig. 2). The stacking sequence for the  $I10 \times 10 \times 0.635$  cm.  $I15 \times 15 \times 0.635$ cm.  $I15 \times 15 \times 0.953$ cm.  $I20 \times 20 \times 0.953$  cm and  $I20 \times 10 \times 0.953$  cm beams is given by the manufacturer.<sup>16</sup> In Fig. 3 the stacking sequence for the  $115 \times 15 \times 0.635$  cm is presented. Classical Lamination Theory<sup>13</sup> provides all the stiffness properties for flanges and webs that need to be considered in the analysis. The beams have been analyzed for various lengths and cross-section geometry. For the web we allow a cubic-shape distortion and for the flanges we do not allow any distortion.

A parametric study of the critical buckling load as a function of lamination angle is also carried out, where the continuous strand mat (CSM) layers of the commercial shapes<sup>16</sup> have been replaced by angle-ply layers  $(+\theta, -\theta)$  to investigate if this lay-up, which is possible to manufacture by pultrusion, gives better lateral-torsional buckling resistance. Bending-twisting coupling terms now play a significant role in the analysis.

# I-beam subjected to uniform moment $M_0$ at the ends

The prebuckling solution can be easily derived for this case using the Laminated Beam Theory.<sup>14</sup> The beams in our case are symmetric and the



Fig. 2. Loading conditions.



Fig. 3. Lay-up for the flange of the  $115 \times 15 \times 0.635$  cm I-beam.

membrane forces are

$$(N_{x}^{f})_{T} = -(N_{x}^{f})_{B} = -\frac{M_{o}}{b_{w}b_{f}t_{f} + \frac{b_{w}^{2}t_{w}}{6}}$$
(24)  
$$N_{w}^{w} = \frac{2M_{o}}{b_{w}b_{f}t_{f} + \frac{b_{w}^{2}t_{w}}{6}} \left(\frac{1}{2} - \frac{y}{b}\right), \quad N_{y}^{w} = 0, \quad N_{xy}^{w} = 0$$

### I-beam subjected to transverse load $P_0$ at the centroid

In this case the membrane forces are

$$N_{x}^{0} = \frac{Pa}{4T}, \quad T = b_{t}b_{w} + \frac{b_{w}^{2}}{6}$$

$$(N_{x}^{t})_{T} = -(N_{x}^{t})_{B} = -2N_{x}^{0}\left(\frac{x}{a}\right) \quad \text{for} \quad 0 < x < \frac{a}{2}$$

$$(N_{x}^{t})_{T} = -(N_{x}^{t})_{B} = 2N_{x}^{0}\left(1 - \frac{x}{a}\right) \quad \text{for} \quad \frac{a}{2} < x < a$$

$$N_{x}^{w} = -4N_{x}^{0}\left(\frac{x}{a}\right)\left(0.5 - \frac{y}{b}\right) \quad \text{for} \quad 0 < x < \frac{a}{2}$$

$$N_{xy}^{w} = -4N_{x}^{0}\left(1 - \frac{x}{a}\right)\left(0.5 - \frac{y}{b}\right) \quad \text{for} \quad \frac{a}{2} < x < a$$

$$N_{xy}^{w} = \frac{P}{2b_{w}} \quad \text{for} \quad 0 < x < \frac{a}{2},$$

$$N_{xy}^{w} = -\frac{P}{2b_{w}} \quad \text{for} \quad \frac{a}{2} < x < a$$

$$N_{xy}^{w} = 0 \quad (25)$$

Figures 4a and 4b show the critical load ratios  $(P_{\rm o}/P_{\rm L})$  and  $(M_{\rm o}/M_{\rm L})$  plotted versus the ratio l/r, where l is the span, r is the radius of gyration of the cross-section,  $P_0$  and  $M_0$  are the critical load and moment for lateral-distortional buckling (Fig. 2), and  $P_{\rm L}$  and  $M_{\rm L}$  are the load and moment that produce flange local buckling only.4,5 The critical load ratio decreases for high aspect ratios l/r and lateral buckling becomes the governing mode of instability, while for shorter beams the buckling mode is distortional.

 $N_{v}^{w} = 0$ 

In Figs 5a and 5b, the critical load ratios are plotted versus the width-height ratio  $b_f/b_w$  (Fig. 1)



Critical load ratio  $P_0/P_1$  versus ratio l/r for load Fig. 4a. applied at the centroid.



Fig. 4b. Critical load ratio  $M_0/M_L$  versus ratio l/r for moment couple applied at the ends.

for constant length. More specifically, the flange which  $b_{\rm f}$  is varied while the height of the web  $b_{\rm w}$ remains constant. One can see that for low width-height ratio (Figs 5a and 5b) there is a reduction in the critical load ratio due to an increased local buckling strength  $P_{\rm L}$  and a reduced lateral buckling strength Po. In Figs 6a and 6b, the critical load ratios are plotted versus the height-width ratio  $b_{\rm w}/b_{\rm f}$  and for constant length. In this case, the height of the web increases while the width of the flange remains constant. It is shown that as the height of the web increases. the buckling strength becomes higher and the mode changes from pure lateral buckling to distortional buckling.

Figure 7 shows the finite difference solution by Mottram<sup>10</sup> (curve 7 on Fig. 2 of Ref. 10), the



Fig. 5a. Critical load ratio  $P_0/P_L$  versus ratio  $b_f/b_w$  for load applied at the centroid and fixed web depth  $b_w$ .



**Fig. 5b.** Critical load ratio  $M_o/M_L$  versus ratio  $b_f/b_w$  for moment couple applied at the ends and fixed web depth  $b_w$ .



**Fig. 6a.** Critical load ratio  $P_0/P_L$  versus ratio  $b_w/b_f$  for load applied at the centroid and fixed flange width  $b_f$ .



**Fig. 6b.** Critical load ratio  $M_o/M_L$  versus ratio  $b_w/b_f$  for moment couple applied at the ends and fixed flange width  $b_f$ .



Fig. 7. Finite difference solution<sup>9</sup> and experimental results (minimum, maximum and average)<sup>9</sup> compared to distortional solution (present).

experimental data of Ref. 10 (average, minimum and maximum values of Table 4 of Ref. 10) for the case of a load applied at the top flange, free warping and fixed rotation at both ends, and the present solution that includes shear deformation and distortion of the web. As expected, the distortional theory with shear deformation predicts lower buckling loads.

To investigate the effects of angle-ply fiber systems on buckling behavior, the CSM layers (Fig. 3) were replaced by  $(+\theta/-\theta)$  layers. Since it is not possible to predict the value of fiber volume fraction achievable in pultrusion before actual production, three values (15%, 30% and 50%) were used in the analysis. The fiber volume fraction on the remaining roving layers (Fig. 3) is kept unchanged. Figures 8a and 8b show the critical load ratios plotted versus the lamination angle  $(+\theta, -\theta)$ . A significant increase of the buckling load ratio can be observed for  $\theta = +30/-30$  with the exception of the case of low fiber volume fraction (15%) and concentrated load at the centroid.

### CONCLUSIONS

For the thin-walled pultruted I-beams considered, coupling of local and lateral buckling modes



**Fig. 8a.** Critical load ratio  $P_o/P_L$  versus the lamination angle  $(+\theta, -\theta)$  for load applied at the centroid.



**Fig. 8b.** Critical load ratio  $M_o/M_L$  versus the lamination angle  $(+\theta, -\theta)$  for moment couple applied at the ends.

always occurs due to the low stiffness of the material in the direction perpendicular to the axis of the beam. For I-beams with high depth-width ratios, lateral buckling is the governing mode of instability. For low height-width ratios, coupled local and distortional buckling results in a reduction of the critical load compared to pure local, or pure lateral buckling loads.

The fiber volume fraction is of significant importance for the critical buckling load determination because higher fiber volume fraction leads directly to higher critical buckling loads and hence to more stable structural members, but the optimum angle is independent of the fiber volume fraction, with few exceptions. For the cases of differently oriented layers  $(+\theta, -\theta)$ , there is a significant increase in the critical buckling load for an optimum angle of  $\theta = +30/-30$ , with the exception of the case of low fiber volume fraction (15%) and concentrated load at the centroid.

Although only the specific cases of simply supported beams with free warping at both ends and concentrated load applied at the centroid or moments applied at the ends have been considered, the theoretical formulation and the solution methodology presented herein can be easily applied for any case of boundary conditions or loading by simply choosing a different shape function and/or computing a new set of prebuckling membrane forces.

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