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Lateral and distortional buckling of pultruded I-beams

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The elastic buckling modes of pultruded I-beams subjected to various loading conditions are studied. The coupling of lateral and distortional buckling, very likely to appear in thin-walled cross-sections, is investigated. Plate theory is used to allow for distortion of the cross-section. Shear effects and bending–twisting coupling are accounted for in the analysis, because of their significant role. The effect of fiber orientation in the matrix and volume fraction is investigated by a parametric study. Pultruded cross-sections are always thin-walled due to constraints in the manufacturing process. Hence, the buckling strength determines the overall strength of the member. The results presented correspond to actual pultruded cross-sections being used in civil engineering types of structure.

INTRODUCTION

Various pultrusion manufacturers produce on an industrial basis beams with a variety of cross-sectional shapes and dimensions (e.g. I-beams, wide flange I-beams, box-beams, angle beams, tubes, etc.). These products are made from polymers (usually called resin in the uncured state and matrix in the cured state) with fiber reinforcement. Polyester, vinylester or epoxy are used as a matrix to hold together E-glass, S-glass, Kevlar or carbon fibers used as reinforcement. Fibers and polymer are joined through the pultrusion process to form the desired cross-section. The present study is concerned with the stability of I-beams and wide-flange I-beams under bending loads.

An I-beam can buckle with various modes depending on the geometry of the cross-section, the material properties, and the boundary and loading conditions. The beam can buckle either locally, or laterally, or with a combination of local and lateral modes. Local buckling is defined as the instability mode when changes in the geometry of the cross-section occur, but not accompanied by lateral displacement or twist. Each part of the cross-section (flanges and web) may buckle as a plate. The coupling of the local buckling of flanges and web may also occur. In the case of lateral buckling, there is a lateral displacement and twist of the cross-section without local changes in the cross-section geometry.

The most general case is when coupling between local modes and lateral modes of buckling occurs. This is called distortional buckling and in many cases of beams with certain dimensions, distortional buckling can result in a significant lowering of the critical load.[1–3]

Many studies have been done separately on the local buckling[4–9] and lateral buckling of I-beams, but only a few on a combination of the buckling modes (distortional buckling).[3] The material used in previous studies on lateral and distortional buckling is steel (homogeneous and isotropic) with the exception of Mottram[10,11] who considered lateral buckling of an orthotropic material. Pultruded cross-sections are thin-walled and each part of the cross-section is treated in this work as a laminated plate. Each lamina can be either specially orthotropic or generally orthotropic. The stiffness coupling terms are important, especially when bending–twisting coupling terms are present because they produce higher instability. The results presented correspond to existing pultruted cross-sections.

THEORETICAL APPROACH

The energy criterion for equilibrium of a structure is that the first variation of the total potential energy is zero

$$\delta V=0 \tag{1}$$

The state of equilibrium when the system loses its stability is characterized by the vanishing of the second variation of the total potential energy[12]

$$\delta^2 V=0 \tag{2}$$

As a first step, we formulate the total potential energy of the system

$$V=\frac{1}{2}\int_V \sigma_{ij}\varepsilon_{ij}\,\mathrm{d}V-\sum_k P_k q_k \tag{3}$$

where P_k are the externally applied forces and q_k are the corresponding displacements.

Considering first the energy terms corresponding to the web for the coordinate system shown in Fig. 1, we use the von-Karman non-linear strains in terms of displacements to describe the kinematics of the system

$$\varepsilon_x=\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2$$

$$\varepsilon_y=\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \tag{4}$$

$$\gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+2\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}$$

The constitutive law for the web considered as a layered plate (laminate) is given from the Classical Lamination Theory[13] as

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & Q_{16}\\ Q_{12} & Q_{22} & Q_{26}\\ Q_{16} & Q_{26} & Q_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix} \tag{5}$$

where Q_{ij} are 'rotated' stiffness quantities corresponding to the global coordinate system (Fig. 1).

Substituting eqns (4) and (5) into eqn (3) we get an expression for the total potential energy for the web in terms of displacements. The second variation of the total potential energy for the web is obtained by performing the following substitutions (eqn (6) into eqn (3))

$$u\to u+\delta u$$

$$v\to v+\delta v \tag{6}$$

$$w\to w+\delta w$$

and collecting the second order terms. The membrane forces N_x, N_y and N_{xy} can be expressed in terms of strains as follows

$$\begin{bmatrix}N_x\\ N_y\\ N_{xy}\end{bmatrix}=\begin{bmatrix}A_{11} & A_{12} & A_{16}\\ A_{12} & A_{22} & A_{26}\\ A_{16} & A_{26} & A_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix} \tag{7}$$

Similarly, the bending and twisting moments M_x, M_y and M_{xy} can be expressed as

$$\begin{bmatrix}M_x\\ M_y\\ M_{xy}\end{bmatrix}=\begin{bmatrix}D_{11} & D_{12} & D_{16}\\ D_{12} & D_{22} & D_{26}\\ D_{16} & D_{26} & D_{66}\end{bmatrix}\begin{bmatrix}\kappa_x\\ \kappa_y\\ \kappa_{xy}\end{bmatrix} \tag{8}$$

where A_{ij} and D_{ij} are the extensional stiffness matrix and the bending stiffness matrix respectively. These matrices can be derived using the Classical Lamination Theory[13] for a layered plate. For an N-layered laminated plate the stiffness terms are given by

$$A_{ij}=\sum_{k=1}^{N}(Q_{ij})_k t_k$$

$$D_{ij}=\sum_{k=1}^{N}(Q_{ij})_k\left(z_k^2 t_k+\frac{t_k^3}{12}\right) \tag{9}$$

Performing the integration over the thickness and identifying the terms corresponding to the extensional stiffness A_{ij} and bending stiffness D_{ij}, we obtain the following expression for the second variation of the total potential energy for the web

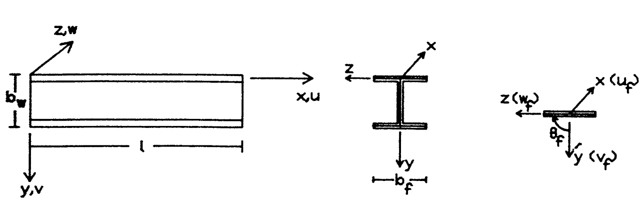

Fig. 1. Coordinate system and geometry of the cross-section. (a) Coordinate system, (b) cross-section, (c) flange.