Evaluation of Finite-Element Calculations in a Part-Circular Crack by Coherent Optics Techniques

by G.H. Kaufmann, A.M. Lopergolo, S.R. Idelsohn and E.J. Barbero

ABSTRACT—Two techniques, speckle photography and holographic interferometry, were used to test three-dimensional finite-element calculations in an internally pressurized cylinder with an external part-circular crack. Opening displacements along the crack line were measured by speckle photography. Radial displacements were obtained from holographic fringe patterns. Good agreement between experimental and numerical data is obtained. Stress-intensity factor variations along the crack front are calculated from numerical results.

Introduction

Engineering estimates for stress-intensity factors in cracked cylinders under internal pressure have been transformed in a very important problem in fracture mechanics. Failure of nuclear piping systems subjected to a cyclic load condition has been traced to fatigue growth of surface cracks. The stress-intensity factor is required to predict the rate of fatigue crack growth for a crack shape which changes continuously because of its growth. For this reason extensive work has been recently done to calculate stress-intensity factors for various shapes of semi-elliptical and semicircular surface cracks in pressurized cylinders. Three-dimensional numerical analysis using the boundary-integral equation method is reported by Heliot et al.1 Atluri and Kathiresan,2 Miyazaki et al.3 and Raju and Newman⁴ use three-dimensional finite-element methods. Variations of the stress-intensity factor along the crack front have been obtained for a range of crack shapes and sizes for external and internal cracks. In spite of the considerable importance of the experimental evaluation of numerical methods, only a few works are available in the literature.

In the past few years, coherent optics techniques such as holographic interferometry and speckle photography have been used to determine the displacement fields adjacent to the crack tip.⁵⁻⁹ Several authors use these measurements to calculate stress-intensity factors for different types of cracks. The use of both techniques in fracture mechanics offers several advantages. As other optical methods, they are noncontact techniques and the model surface needs no preparation. They do not require materials with special optical characteristics, so they can be directly applied to any opaque materials such as metals or composites.

The purpose of this paper is to show the usefulness of both techniques in fracture mechanics to evaluate calculations obtained by means of numerical methods. As an example, the case of an external surface part-circular crack in a thick cylinder is analyzed. Finite-element calculations are performed; good agreement is found between experimental and numerical results. Variations of the stress-intensity factor along the crack front calculated by the finite-element idealization are compared with results by Raju and Newman.⁴

Experimental Test

Specimen

The circular cylinder used in this study was made of PMMA. This material was chosen because it is a brittle one; so no plastic flow was expected around the crack tip. Hence, experimental results can be compared with those calculated using linear-elastic fracture mechanics. The cylinder had a wall thickness t of 0.5 cm. Its internal radius R_i was 2.98 cm. The thickness to radius ratio t/R_i was 0.168. Its length 2b was 20 cm, with a 1-cm clamping on both edges, as shown in Fig. 1.

After the cylinder was cut into size, a central axial partcircular slot was inserted on the external surface of the specimen with a circular saw. The crack length 2c was 1.36 cm. The crack depth *a* was 0.23 cm, which gave an a/c ratio of 0.338 and an a/t ratio of 0.46. The slot was 0.02-cm wide and terminated in a vee notch with a very small radius.

Next the specimen was heat treated to relieve stresses. Afterwards, the area surrounding the cracks was sprayed with a thin coat of matt-white paint.

Two square-end steel plates were made to clamp the edges of the cylinder. On each end-plate there was a circular groove, 1-cm deep, which fitted nicely with the cylinder. Before the two ends of the specimen were slid into the groove, a cyanoacrylate cement was poured, thus clamping the ends of the cylinder to the end-plates. Four rods parallel to the cylinder and fastened to both endplates made sure the clamping would not become unglued. Pressure was applied by means of an hydraulic system measured with a manometer. A valve was used to maintain the pressure at a constant value during the exposure time.

G.H. Kaufmann is Professor, and A.M. Lopergolo is Research Fellow, Instituto de Fisica de Rosario (CONICET-UNR), Avda, Pellegrini 250, 2000 Rosario, Argentina. S.R. Idelsohn is Professor, and E.J. Barbero is Research Fellow, Instituto de Desarrollo Tecnológico para la Industria Química (CONICET-UNL), P.O. Box 91, 3000 Santa Fe, Argentina.

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Values of the elastic constant, E = 2920 MPa and $\nu = 0.33$, were obtained from a calibration of a bar of the same material using electrical strain gages.

Speckle Photography

The speckle-photography technique¹⁰ was used to measure the crack opening (in-plane) displacements u_{θ} along the slot line. They were obtained by measuring the crack aperture along two lines parallel to the slot direction, one at each side of the crack and at a distance of 0.025 cm from it. The u_{θ} displacements were calculated as one half of the difference between the cracks apertures corresponding to a pair of points at each side of the slot.

The sensitivity of this technique is limited by the average speckle size. It can be shown that displacements to be measured must be greater than¹⁰

$$\delta_o = 1.22 \ \lambda F \left(1 + m\right) / m \tag{1}$$

where λ is the wavelength of the laser, F is the aperture ratio of the recording optical system and m is its magnification. Nevertheless displacements observed along the crack line were very small (about a few microns) which were outside the sensitivity of the technique given by eq (1). This could be increased, however, by superimposing a small known artificial displacement. For our application, a rigid-body displacement of about three times the speckle diameter was produced by mounting the photographic camera on a vernier table. This displacement was cancelled when the difference between the crack apertures corresponding to a pair of opposite points along the slot was done.

When the cylinder was pressurized between both exposures, its surface moved in the radial direction. It is well known that out-of-plane displacements produce speckle decorrelation which causes a decrease in fringe contrast and introduces errors in the measurements.¹⁰ The Rayleigh criterion establishes the movement which can be tolerated without any appreciable degradation of the fringes. This movement must be less than

$$\Delta = 4\lambda F^2 (1+m)^2 / m^2 \tag{2}$$

For the experiments presented in this paper $\Delta \sim 500 \ \mu m$. Taking into account that the radial expansion for the maximum pressure utilized was about 30 μm , fringes with good contrast were obtained without using any compensation procedure. Figure 2 shows a typical fringe pattern.

To simplify data reduction, the cylinder surface was imaged on to a Kodak SO-253 photographic plate mounted parallel to the specimen by means of a well-corrected camera lens. A 21-cm focal length of F = 8 aperture was used at m = 2.8 magnification. The specimen was illuminated by an He-Ne laser. Double-exposure speckle-grams were analyzed using the pointwise technique. By measuring the spacing and inclination of the fringes, aperture displacements along the crack line were calculated.

Holographic Interferometry

Holographic interferometry was used to measure the radial (out-of-plane) displacement u_r along the crack line. The usual holographic setup to measure out-of-plane displacements was utilized.¹¹ The cylinder was normally illuminated with a collimated beam of He-N_e laser light. It was mounted parallel to the holographic plate. Several

double-exposure holograms were recorded on Agfa-Gevaert 8E75 plates for different pressures. After processing the hologram, the image showing deformation fringes was reconstructed and photographed at normal incidence. Under these conditions, any point on the axial crack line has a radial displacement given by¹¹

$$u_r = N \lambda/2 \tag{3}$$

where N takes on integer values at the center of each





Fig. 2-Typical speckle fringe pattern

bright fringe. Figure 3 shows a typical fringe pattern. From each pattern, the change in radial displacement along the crack line was obtained and was compared with numerical predictions.

Numerical Analysis

Three-dimensional finite-element calculations were performed so as to be compared with the experimental results. Figure 4 shows the discretization used. It employs 275 elements with 4107 degrees of freedom. Two types of elements were used to model the cylindrical vessel: a layer of collapsed singular elements12 around the crack front and 20-node isoparametric elements with 60 degrees of freedom elsewhere. The curvature of the model was represented through second-grade parabolas. Threedimensional elements were used because the model was a thick cylinder. The dimensions of the elements in the neighborhood of the crack were determined by means of a convergence study on a known plane-strain crack problem. As shown by Ref. 12, collapsed singular elements are the best choice among the singular isoparametric threedimensional elements to model the singular crack problem.



Fig. 3—Typical interference fringes

The model idealizes one-eighth of the vessel, so that it simulates a vessel with two diametrically located surface cracks, 180-degrees apart. As is shown by Ref. 4, this does not introduce appreciable error.

Symmetry boundary conditions were applied on the x = 0, y = 0, z = 0 planes, and clamping restrictions on the z = b plane. Internal pressure was introduced by means of a system of energetically equivalent radial forces. The computer program SAMCEF¹³ was used for the analysis.

Mode-I elastic-stress-intensity factors were calculated by means of the crack-opening displacement method, assuming plane-strain conditions.

Results

Opening displacement along the crack line for an applied pressure p = 1.2 MPa are shown in Fig. 5. Radial displacement along the same line for p = 0.2 MPa are plotted in Fig. 6. Satisfactory agreement between experimental and numerical results is obtained. The disagreement of data with the numerical solution at distances far from the crack tip could be due to the fact that the plane-strain condition is lost at those distances. Furthermore, it must be pointed out that displacements were measured over the free surface and so measurements are neither plane strain nor plane stress.

Figure 7 shows the distribution of nondimensional stress-intensity factors along the crack front obtained from numerical calculations. The results are normalized by the stress-intensity factor K_o at $\phi = 90$ degrees for an elliptical crack embedded in an infinite body subjected to a uniform tension stress σ_o ,

$$K_o = \sigma_o \sqrt{\frac{\pi a}{Q}}$$

with

$$\sigma_o = \frac{2R_i^2}{R_o^2 - R_i^2} P$$

where R_o is the external radius of the cylinder, and Q is the square of the complete elliptic integral of second kind approximated by $Q = 1 + 1.464 (a/c)^{1.65}$. The results of Raju and Newman⁴ for a semielliptical crack of t/R =



Fig. 4-Finite-element mesh

0.1, a/c = 0.4 and a/t = 0.5 and the results from ASME Boiler & Pressure Vessel Code Section XI are also presented for comparison in Fig. 7.

Conclusion

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Concluding, the use of speckle-photography and holographic-interferometry techniques for the measurement of displacements surrounding a crack tip in an internally pressurized cylinder was successfully demonstrated. This work also demonstrates the practicability of





using both coherent-optics techniques to evaluate calculations obtained by means of numerical methods, such as finite elements.

References

I. Heliot, J., Lahbens, R.C. and Pellissier-Tanon, A., "Semi-Elliptical Cracks in the Meridional Plane of a Cylinder Subjected to Stress Gradients," Fracture Mechanics, ASTM STP 677, Amer. Soc. Test. and Mat., 341-364 (1979).

2. Atluri, S.N. and Kathiresan, K., "3 D Analysis of Surface Flaws in Thick-Walled Reactor Pressure Vessels Using a Displacement-Hybrid Finite Element Method," Nucl. Eng. Des., 51, 163-176 (1979).

3. Miyazaki, N., Shibata, K., Watanabe, T. and Tagata, K., "Stress Intensity Factor Analysis of Surface Cracks in Three-dimensional Structures - Comparison of the Finite Element Solutions with the Results Obtained by the Simplified Estimation Methods," Int. J. Pres. Ves. & Piping, 15, 37-59 (1984).

4. Raju, I.S. and Newman, J.C., "Stress-Intensity Factors for Internal and External Surface Cracks in Cylindrical Vessels," J. Pres. Ves. Tech., Trans. ASME, 104, 293-298 (1982).

5. Barker, D.B. and Fourney, M.E., "Displacement Measurements in the Interior of Three-dimensional Bodies Using Scattered-light Speckle Patterns," EXPERIMENTAL MECHANICS, 16, 209-214 (1976).

6. Chiang, F.P. and Asundi, A., "A White Light Speckle Method Applied to the Determination of Stress-Intensity Factor and Displacement Field Around a Crack Tip," Eng. Fract. Mech., 15, 115-121 (1981).

7. Kaufmann, G.H., Lopergolo, A.M. and Idelsohn, S., "Evaluation of Finite Element Calculations in a Cracked Cylinder under Internal Pressure by Speckle Photography," J. Appl. Mech., Trans. ASME, 50, 896-897 (1983).

8. Hsu, T.R., Lewak, R. and Wilkins, B.J.S., "Measurements of Crack Growth in a Solid at Elevated Temperature by Holographic Interferometry," EXPERIMENTAL MECHANICS, 18, 297-302 (1978). 9. Ennos, A.E. and Virdee, M.S., "Application of Reflection

9. Ennos, A.E. and Virdee, M.S., "Application of Reflection Holography to Deformation Measurement Problems," EXPERIMENTAL MECHANICS, 22, 202-209 (1982).

10. Erf, R.K., ed., Speckle Metrology, Academic Press (1978).

11. Vest, C.M., Holographic Interferometry, John Wiley and Sons (1979).

12. Barsoum, R.S., "Triangular Quarter-Point Elements as Elastic and Perfectly-Plastic Crack Tip Elements," Int. J. Num. Meth. in Eng., 11, 85-98 (1977).

13. SAMCEF, Systeme d'Analyse des Milieux Continus par Elements Finis, Manuel d'Utilisation, Universite de Liege.





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