# Contents

**Preface** xv

**Author Bio** xix

**Acknowledgments** xxi

**List of Symbols and Acronyms** xxiii

**List of Examples** xxxiii

**Errata** xxxv

**1 Mechanics of Orthotropic Materials** 1
- 1.1 Lamina Coordinate System . . . 1
- 1.2 Displacements . . . 1
- 1.3 Strain . . . 2
- 1.4 Stress . . . 3
- 1.5 Contracted Notation . . . 4
  - 1.5.1 Alternate Contracted Notation . . . 5
- 1.6 Equilibrium and Virtual Work . . . 6
- 1.7 Boundary Conditions . . . 8
  - 1.7.1 Traction Boundary Conditions . . . 8
  - 1.7.2 Free Surface Boundary Conditions . . . 8
- 1.8 Continuity Conditions . . . 9
  - 1.8.1 Traction Continuity . . . 9
  - 1.8.2 Displacement Continuity . . . 9
- 1.9 Compatibility . . . 10
- 1.10 Coordinate Transformations . . . 10
  - 1.10.1 Stress Transformation . . . 13
  - 1.10.2 Strain Transformation . . . 15
- 1.11 Transformation of Constitutive Equations . . . 16
- 1.12 3D Constitutive Equations . . . 17
  - 1.12.1 Anisotropic Material . . . 18
  - 1.12.2 Monoclinic Material . . . 19
  - 1.12.3 Orthotropic Material . . . 20

1.12.4 Transversely Isotropic Material . . . 22
1.12.5 Isotropic Material . . . 23
1.13 Engineering Constants . . . 24
1.13.1 Restrictions on Engineering Constants . . . 28
1.14 From 3D to Plane Stress Equations . . . 29
1.15 Apparent Laminate Properties . . . 31
Suggested Problems . . . 32

**2 Introduction to Finite Element Analysis 35**
2.1 Basic FEM Procedure . . . 35
2.1.1 Discretization . . . 36
2.1.2 Element Equations . . . 36
2.1.3 Approximation over an Element . . . 37
2.1.4 Interpolation Functions . . . 38
2.1.5 Element Equations for a Specific Problem . . . 40
2.1.6 Assembly of Element Equations . . . 41
2.1.7 Boundary Conditions . . . 42
2.1.8 Solution of the Equations . . . 42
2.1.9 Solution Inside the Elements . . . 42
2.1.10 Derived Results . . . 42
2.2 General Finite Element Procedure . . . 43
2.3 Solid Modeling, Analysis, and Visualization . . . 46
2.3.1 Interacting with Abaqus . . . 47
2.3.2 Model Geometry . . . 48
2.3.3 Manual Meshing . . . 49
2.3.4 Solid Modeling . . . 53
2.3.5 The CAE Window . . . 53
2.3.6 Material and Section Properties . . . 58
2.3.7 Assembly . . . 62
2.3.8 Solution Steps . . . 62
2.3.9 Loads . . . 65
2.3.10 Boundary Conditions . . . 65
2.3.11 Meshing and Element Type . . . 68
2.3.12 Solution Phase . . . 73
2.3.13 Post-processing and Visualization . . . 74
2.4 Interactions and Constraints . . . 91
2.4.1 Tie Constraint . . . 91
2.4.2 Rigid Body Constraint . . . 92
2.4.3 Display Body . . . 93
2.4.4 Coupling . . . 93
2.4.5 Adjust Points . . . 95
2.4.6 MPC Constraint . . . 95
2.4.7 Shell-to-solid Coupling . . . 95
2.4.8 Embedded Region . . . 95

2.4.9 Multi-point Constraint Equations . . . 95
Suggested Problems . . . 95

**3 Elasticity and Strength of Laminates 97**
3.1 Kinematic of Shells . . . 98
3.1.1 First-Order Shear Deformation Theory . . . 99
3.1.2 Kirchhoff Theory . . . 104
3.1.3 Simply Supported Boundary Conditions . . . 105
3.2 Finite Element Analysis of Laminates . . . 106
3.2.1 Element Types and Naming Convention . . . 107
3.2.2 Thin (Kirchhoff) Shell Elements . . . 110
3.2.3 Thick Shell Elements . . . 111
3.2.4 General-purpose (FSDT) Shell Elements . . . 111
3.2.5 Continuum Shell Elements . . . 111
3.2.6 Sandwich Shells . . . 112
3.2.7 Nodes and Curvature . . . 112
3.2.8 Drilling Rotation . . . 112
3.2.9 A, B, D, H Input Data for Laminate FEA . . . 113
3.2.10 Equivalent Orthotropic Input for Laminate FEA . . . 119
3.2.11 LSS for Multi-directional Laminate FEA . . . 125
3.2.12 FEA of Ply Drop-Off Laminates . . . 135
3.2.13 FEA of Sandwich Shells . . . 144
3.2.14 Element Coordinate System . . . 156
3.3 Failure Criteria . . . 169
3.3.1 2D Failure Criteria . . . 169
3.3.2 3D Failure Criteria . . . 172
3.4 Predefined Fields . . . 175
Suggested Problems . . . 177

**4 Buckling 181**
4.1 Eigenvalue Buckling Analysis . . . 181
4.1.1 Imperfection Sensitivity . . . 187
4.1.2 Asymmetric Bifurcation . . . 187
4.1.3 Post-critical Path . . . 188
4.2 Continuation Methods . . . 192
Suggested Problems . . . 197

**5 Free Edge Stresses 199**
5.1 Poisson's Mismatch . . . 200
5.1.1 Interlaminar Force . . . 200
5.1.2 Interlaminar Moment . . . 201
5.2 Coefficient of Mutual Influence . . . 208
5.2.1 Interlaminar Stress due to Mutual Influence . . . 211
Suggested Problems . . . 216

**6 Computational Micromechanics 221**
6.1 Analytical Homogenization . . . 222
6.1.1 Reuss Model . . . 222
6.1.2 Voigt Model . . . 223
6.1.3 Periodic Microstructure Micromechanics . . . 223
6.1.4 Transversely Isotropic Averaging . . . 224
6.2 Numerical Homogenization . . . 226
6.3 Local-Global Analysis . . . 247
6.4 Laminated RVE . . . 251
Suggested Problems . . . 256

**7 Viscoelasticity 257**
7.1 Viscoelastic Models . . . 259
7.1.1 Maxwell Model . . . 259
7.1.2 Kelvin Model . . . 260
7.1.3 Standard Linear Solid . . . 261
7.1.4 Maxwell-Kelvin Model . . . 261
7.1.5 Power Law . . . 262
7.1.6 Prony Series . . . 262
7.1.7 Standard Nonlinear Solid . . . 264
7.1.8 Nonlinear Power Law . . . 264
7.2 Boltzmann Superposition . . . 266
7.2.1 Linear Viscoelastic Material . . . 266
7.2.2 Un-aging Viscoelastic Material . . . 267
7.3 Correspondence Principle . . . 268
7.4 Frequency Domain . . . 269
7.5 Spectrum Representation . . . 270
7.6 Micromechanics of Viscoelastic Composites . . . 270
7.6.1 One-Dimensional Case . . . 270
7.6.2 Three-Dimensional Case . . . 272
7.7 Macromechanics of Viscoelastic Composites . . . 277
7.7.1 Balanced Symmetric Laminates . . . 277
7.7.2 General Laminates . . . 277
7.8 FEA of Viscoelastic Composites . . . 277
Suggested Problems . . . 290

**8 Continuum Damage Mechanics 293**
8.1 One-Dimensional Damage Mechanics . . . 294
8.1.1 Damage Variable . . . 294
8.1.2 Damage Threshold and Activation Function . . . 296
8.1.3 Kinetic Equation . . . 297
8.1.4 Statistical Interpretation of the Kinetic Equation . . . 298
8.1.5 One-Dimensional Random-Strength Model . . . 299
8.1.6 Fiber Direction, Tension Damage . . . 304
8.1.7 Fiber Direction, Compression Damage . . . 310

8.2 Multidimensional Damage and Effective Spaces . . . 314
8.3 Thermodynamics Formulation . . . 315
8.3.1 First Law . . . 316
8.3.2 Second Law . . . 317
8.4 Kinetic Law in Three-Dimensional Space . . . 323
8.4.1 Return-Mapping Algorithm . . . 326
8.5 Damage and Plasticity . . . 332
Suggested Problems . . . 334

**9 Discrete Damage Mechanics 337**
9.1 Overview . . . 338
9.2 Approximations . . . 342
9.3 Lamina Constitutive Equation . . . 343
9.4 Displacement Field . . . 344
9.4.1 Boundary Conditions for $\Delta T = 0$ . . . 346
9.4.2 Boundary Conditions for $\Delta T \neq 0$ . . . 347
9.5 Degraded Laminate Stiffness and CTE . . . 347
9.6 Degraded Lamina Stiffness . . . 348
9.7 Fracture Energy . . . 349
9.8 Solution Algorithm . . . 350
9.8.1 Lamina Iterations . . . 350
9.8.2 Laminate Iterations . . . 351
Suggested Problems . . . 358

**10 Delaminations 361**
10.1 Cohesive Zone Model . . . 365
10.1.1 Single Mode Cohesive Model . . . 366
10.1.2 Mixed-mode Cohesive Model . . . 370
10.2 Virtual Crack Closure Technique . . . 381
10.3 Determination of CZM Parameters . . . 384
10.3.1 Elastic Loading . . . 385
10.3.2 Softening Behavior . . . 387
10.3.3 LINEAR CZM . . . 389
10.3.4 TABULAR CZM . . . 392
10.3.5 Mixed-mode Parameter . . . 397
10.4 Modeling Considerations . . . 399
10.4.1 Damage Stabilization Cohesive . . . 399
10.4.2 Damping . . . 399
10.4.3 Symmetry . . . 400
10.4.4 ENCASTRE . . . 401
10.4.5 Mesh Refinement . . . 401
10.4.6 Abaqus CAE and Input File . . . 402
10.5 Script for Example 10.4 . . . 403
10.5.1 DCBCZMparams.py . . . 404
Suggested Problems . . . 412

**11 Fatigue** **413**
11.1 Temperature Dependent Properties . . . 413
11.2 The Quasi-Static Problem . . . 415
11.3 The First Cycle of a Fatigue Test . . . 417
11.4 Fatigue Damage Criterion . . . 419
11.5 Master Paris Law . . . 421
11.6 Thermal and Fatigue Damage Prediction . . . 425
11.6.1 Implementation . . . 427
11.7 Thermomechanical Equivalence . . . 434
11.7.1 Damage Equivalence . . . 435
11.7.2 Defect Nucleation Equivalence . . . 435
11.8 Software Implementation . . . 436
11.9 Uniaxial Mechanical Test . . . 437

**12 Abaqus Programmable Features** **443**
12.1 User Materials in Abaqus Standard . . . 443
12.2 UMAT for Linear Elastic Shells . . . 444
12.2.1 User Subroutine Interface . . . 444
12.2.2 State Variables and Constants . . . 445
12.2.3 User's Code . . . 446
12.2.4 UMATS8R5.FOR . . . 447
12.3 UMAT for Orthotropic Viscoelasticity . . . 454
12.3.1 User Subroutine Interface . . . 454
12.3.2 State Variables and Constants . . . 455
12.3.3 User's Code . . . 456
12.3.4 UMAT3DVISCO.FOR . . . 457
12.4 Constraint Equations and Periodicity . . . 459
12.4.1 Ex_6.4.py . . . 459
12.4.2 PBC_2d.py . . . 465
12.4.3 Mesoscale Effective Stress from FEA-RVE Solution . . . 466
12.5 UMAT 1D . . . 468
12.5.1 User Subroutine Interface . . . 468
12.5.2 State Variables and Constants . . . 469
12.5.3 User's Code . . . 469
12.5.4 UMAT1D83.FOR . . . 470
12.6 UMAT Plane Stress . . . 473
12.6.1 User Subroutine Interface . . . 473
12.6.2 State Variables and Constants . . . 473
12.6.3 User's Code . . . 474
12.6.4 UMATPS85.FOR . . . 476
12.7 UGENS. User General Section . . . 477
12.7.1 Alternative Solution for Example 9.1 . . . 477
12.7.2 Alternative Solution for Example 11.1 . . . 481
12.8 Execute Abaqus from MATLAB . . . 489

**A Tie Constraints** **491**

**B Tensor Algebra** **493**
B.1 Principal Directions of Stress and Strain . . . . . 493
B.2 Tensor Symmetry . . . . . 493
B.3 Matrix Representation of a Tensor . . . . . 494
B.4 Double Contraction . . . . . 495
B.5 Tensor Inversion . . . . . 495
B.6 Tensor Differentiation . . . . . 496
B.6.1 Derivative of a Tensor with Respect to Itself . . . . . 496
B.6.2 Derivative of the Inverse of a Tensor Respect to the Tensor . 497

**C Second-Order Diagonal Damage Models** **499**
C.1 Effective and Damaged Spaces . . . . . 499
C.2 Thermodynamic Force $\mathbf{Y}$ . . . . . 500
C.3 Damage Surface . . . . . 502
C.4 Unrecoverable-Strain Surface . . . . . 503

**D Micromechanics** **505**
D.1 Periodic Microstructure Micromechanics . . . . . 505
D.2 Coefficients of Thermal Expansion . . . . . 506

**E Software Used** **509**
E.1 How to Install Abaqus 2020 with Intel oneAPI . . . . . 511
E.2 BMI3 . . . . . 514

**References** **517**

**Index** **531**