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UNIVERSAL KNOCKDOWN FACTORS FOR STRENGTH OF CARBON/EPOXY LAMINATES

Ever J. Barbero
West Virginia University
715 ESB
Morgantown, WV 26505-6106

ABSTRACT

A methodology to estimate universal knockdown factors applicable to Carbon/Epoxy material systems is proposed. Basis values for any material system can be obtained by multiplying the nominal strength of the material system of interest by the universal knockdown factors. The methodology allows one to obtain different knockdown factors for A-, B-, and C- basis, or for any other level of coverage and confidence desired. Experimental data from twelve studies is used to demonstrate the method. The data encompasses a variety of Carbon/Epoxy materials systems, laminates stacking sequences, biaxial load ratios, and experimental techniques.

1. INTRODUCTION

Many material systems are being developed to fill the needs of an ever-increasing range of applications for composite materials. While diversity of material systems provides opportunities for matching the perfect material for each application, it is challenging because it requires full material characterization of every new material before it can be used. Perhaps the most onerous task is the experimental determination of Basis Values. They account for the variability of material properties and thus are indispensable in Design for Reliability. Since a large data set is needed to characterize variability, basis values are expensive to obtain and rare to find. Furthermore, some material tests such as biaxial testing, hot-humid, and cryogenic tests, are difficult, time consuming, and expensive. Testing of multiple specimens is often prohibitive.

On the other hand, numerous studies have been performed at great cost and effort for representative materials systems, such as Carbon/Epoxy systems with a variety of Carbon fibers, Epoxies, laminate stacking sequences (LSS), biaxial load ratios, and test conditions. Therefore, the objective of this work is to find a way to use available test data to estimate the basis values of similar material systems.

Building on [1, 2], a methodology to estimate universal knockdown factors, applicable to all Carbon/Epoxy material systems is proposed and illustrated for the case of biaxial strength of composite laminates. Basis values for a any material system can then be obtained by multiplying the nominal strength of the material of interest by universal knockdown factors. In this way, predicted basis values are representative of the variability introduced by processing and other conditions on similar materials covering the universe of existing data for similar materials systems.

It is hypothesized that the variability of new, still untested materials, should not be worse than that of previously tested material systems if the former are to succeed in the marketplace.

Eventually, to validate the predicted basis values, definite testing of the particular material system can be performed, but at a later stage, closer to prototyping, after preliminary design is completed and material selection is less likely to be affected by design considerations.

The proposed methodology allows the practitioner to obtain different knockdown factors and basis values for A-, B-, and C- basis, or any other level of coverage and confidence desired. Experimental data from twelve studies is used to demonstrate the method.

2. METHODOLOGY

The objective of this section is to present the methodology to calculate universal knockdown factors for strength and to illustrate it with the calculation of A-, B-, and C-basis values. Biaxial tests are reported in the literature for various values of load ratio, defined as

$$R = \frac{N_x}{N_y} = \frac{\sigma_x}{\sigma_y} \tag{1}$$

where N_x, N_y are the in-plane loads per unit width applied to the laminate and σ_x, σ_y are the stress averages over the laminate thickness. A-, B-, and C-basis values correspond to coverage Q and confidence C of 99/95%, 90/95%, and 95/95%, respectively [3, Sect. 1.5.6] but other sets of coverage/confidence values can be calculated as well.

In this work, it is assumed that the maximum strain criterion (MSC) provides a good representation of biaxial failure, as shown in Figure 1, which includes 147 data points from 12 studies using 7 material systems.

For each lamina k in a laminate, the MSC failure index r_k is calculated as

$$r_k = < \frac{\varepsilon_1}{\varepsilon_{1t}} > + < -\frac{\varepsilon_1}{\varepsilon_{1c}} > \tag{2}$$

where ε_1 is the longitudinal strain (along the fiber direction), ε_{1t}, ε_{1c} are the longitudinal tensile and compressive strains-to-failure of the unidirectional (UD) lamina obtained from uniaxial tensile tests, and < > is the Macaulay operator that yields only the positive part of the argument. Then, the failure index for the laminate is simply the maximum of all lamina values

$$r = \max(r_k) \qquad k = \ldots N \tag{3}$$

where N is the number of laminas in the laminate and $r \geq 1$ indicates failure. Some studies report experimental values of laminate strains-to-failure ε_x, ε_y for various values of load ratio. If the biaxial data is available in terms of stress pairs σ_x, σ_y, laminate strains-to-failure are computed using linear elastic constitutive equations. Next, all 147 data pairs ε_x, ε_y are normalized by the strains-to-failure of the UD lamina, whether tensile or compressive, as follows

$$\epsilon_i = < \frac{\varepsilon_i}{\varepsilon_{1t}} > + < -\frac{\varepsilon_i}{\varepsilon_{1c}} > \qquad i = x, y \tag{4}$$

and displayed in Figure 1. It can be seen that the data is clustered around the maximum strain failure envelope, which in Figure 1 is represented by the box defined by values 1,-1, for tension and compression, respectively.

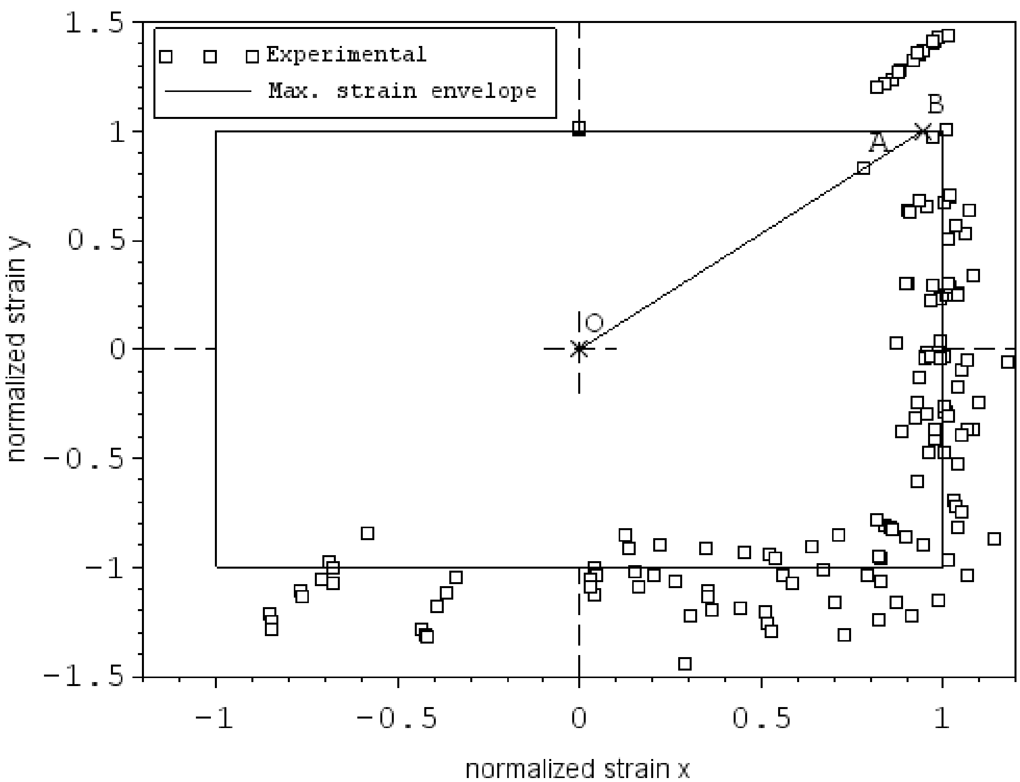

Figure 1. Normalized strains-to-failure data compared to maximum strain envelope. OAB is the load line for data point A.

For each data point, a load line can be drawn that originates at the origin and passes through the data point, eventually intersecting the failure envelope. The failure index r for each data point can be calculated graphically as $r=OA/OB$, where OA is the distance from the origin to the data point, and OB is the distance from the origin to the intersection of the load line with the failure envelope. The discrepancy between the strength data and the prediction of strength provided by the failure envelope can be assessed directly by r, which can be calculated with (2-3) for each data point.

Various data points have different values of r, which are less than one if inside the envelope, and larger than one if outside of it. The set R of all individual r-values is a stochastic variable that can be represented by a probability distribution. A histogram of the data is shown in Figure 2 and Table 1.

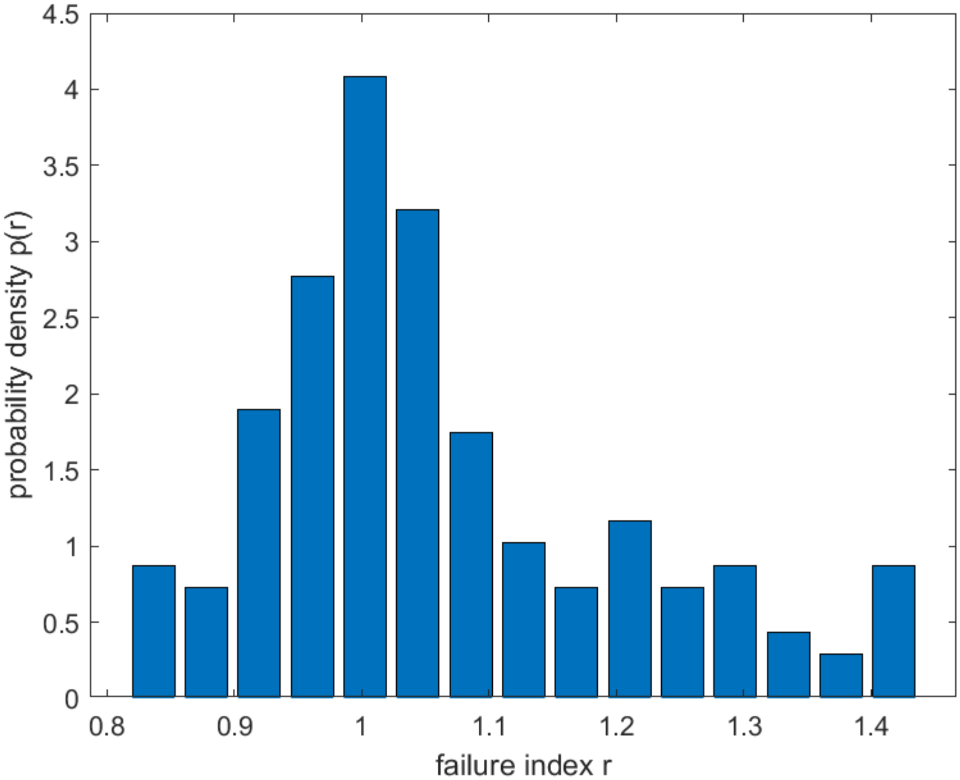

Figure 2. Histogram of failure index events *r* per bin, normalized to total area equal to one.

Table 1: Frequency distribution for bin width 0.467

6	5	13	19	28	22	12	7	5	8	5	6	3	2	6

Next, a probability plot [4] can be used to determine which distribution best fits the data. Among the most popular distributions (Normal, log-normal, and Weibull) the log-normal provides the best fit, as attested by the probability plot shown in Figure 3.

The Log-Normal Distribution assigns zero probability for negative values. This property is useful when, for some physical reason, the values of a certain random variable are known to be strictly positive. For example, the failure index cannot be negative.

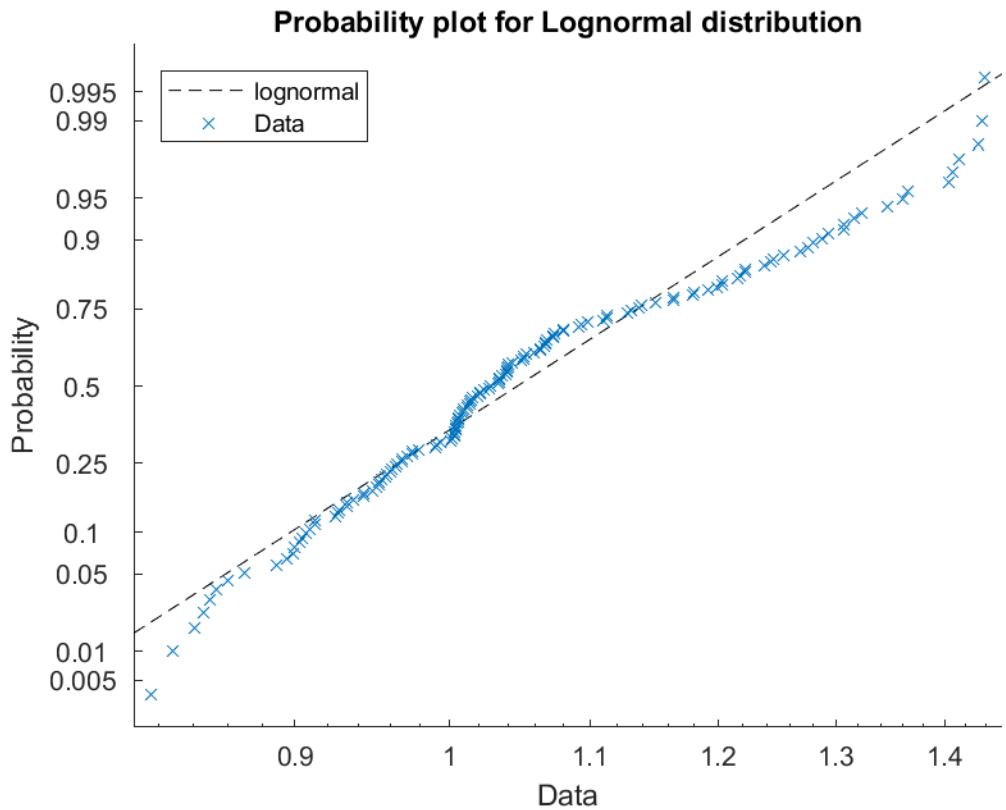

Figure 3. Probability plot displaying the actual probability for each data point and a linear fit with parameters μ_{LN}, σ_{LN} for the log-normal probability function.

The procedure to calculate basis values for a log-normal distributed variable is similar as that for a Normal distribution. First take *log()* of the data, then calculate the mean and standard deviation in log space

$$X = \log(R) \qquad (5)$$
$$\mu_{LN} = \text{mean}(X)$$
$$\sigma_{LN} = \text{std}(X)$$

where μ_{LN} = *mean(log R)* is not the same as *mean(R)*. The former is the mean in *log()* space, and the latter is the mean of the actual data *R*. Then, calculate the basis values for the Normal distribution X using [3, (1.25)] and take the *exp()* of that

$$x_{QC} = \exp\left[\mu_{LN} - k_{QC}(n)\,\sigma_{LN}\right] \qquad (6)$$

where $k_{QC}(n)$ is the coefficient for a Normal distribution, which can be calculated as explained in [3, Sect. 1.5.6] for coverage Q, confidence C, and number of specimens *n*. For *n* = 147 data points (shown in Figure 1) the values of k_{QC} for A- B-, and C-basis are practically the same as for *n* = 100 found in Table [3, Table 1.8], i.e, 2.6840, 1.5267, and 1.9265 respectively. Therefore, using (6), universal basis values x_{QC} for A-, B- and C-basis of the failure index data in R are reported in Table 2.

Table 2: Universal knock down factors for A-, B-, and C-basis of Carbon/Epoxy laminates.

Basis	x_{QC}
A	0.7458
B	0.8659
C	0.8224

Since the x_{QC} values reported in Table 2 are *basis values for normalized failure index data*, they are in effect *knock down factors* that can be used to reduce nominal strength to allowable values, the later satisfying coverage Q with confidence C. Therefore, the actual basis-values for strain-to-failure and strength are calculated as

$$\begin{aligned} \epsilon_{1t}^{basis} &= x_{QC}\,\epsilon_{1t} \quad ; \quad F_{1t}^{basis} = x_{QC}\,F_{1t} \\ \epsilon_{1c}^{basis} &= x_{QC}\,\epsilon_{1c} \quad ; \quad F_{1c}^{basis} = x_{QC}\,F_{1c} \end{aligned} \tag{7}$$

where ε_{1t}, ε_{1c} are *nominal* longitudinal strains-to-failure and F_{1t}, F_{1c} are *nominal* strengths in tension and compression obtained from UD lamina tests. The knock down factors x_{QC} obtained in this work are universal (independent of material system). Actual basis values for a specific material system can be recovered in terms of the nominal values for the UD lamina for the chosen material system as in (7).

While basis values are scarce, nominal values are readily available for most material systems. Both universal x_{QC} and the actual basis values calculated with (7) account for the variability observed in the universe of experimental data available in the literature. As more experimental data becomes available, the universal knock down factors can be refined. Eventually, when the design reaches certain maturity, the designer may always choose to perform the relevant experimentation, however costly, to obtain basis values for a specific material system by direct experimentation.

In Table 3, predicted basis values are compared with measured basis values for specific material systems that were not part of the process that led to the values reported in Table 2. Predicted values are computed using (7). Knock down factors from Table 2 and nominal values of UD tensile/compressive strength F_{1t}, F_{1c} taken form [3, Tables 1.3–4] are used in (7). Results are then compared to measured basis values reported in [5, 6] for similar materials tested at 24°C.

Table 3: Calculated basis value (x_{QC} times nominal F_{1T}) vs. measured basis from [5,6].

Basis (property)	Nominal [5,6]	Universal x_{QC} Table 2	Predicted $x_{QC} F_{1t}$	Measured [5,6]	% error
	T700/2510				
A-basis (F1t)	2172	0.7458	1620	1740	-6.9%
B-basis (F1t)	2172	0.8659	1881	1911	-1.6%
A-basis (F1c)	1450	0.7458	1081	1169	-7.5%
B-basis (F1c)	1450	0.8659	1256	1280	-1.9%
	AS4/3502				
B-basis (F1t)	1779	0.8659	1540	1413	9.0%
B-basis (F1c)	1407	0.8659	1218	1179	3.3%
	AS4/997				
B-basis (F1t)	2255	0.8659	1953	2013	-3.0%
B-basis (F1c)	1579	0.8659	1367	1344	1.7%
	T650-35/976				
B-basis (F1t)	1593	0.8659	1379	1393	-1.0%

3. CONCLUSIONS

For those materials systems for which basis values are not readily available, preliminary design would benefit from estimation of basis values based on the universal knock down factors presented in this study. Comparison between predicted and measured basis values using readily available data suggests reasonable accuracy. Using the proposed methodology, the universal knock down factors can be refined as more experimental data becomes available. Data is shown to support the maximum strain criterion as adequate for estimation of universal values for lamina properties calculated from laminate test data under biaxial load conditions. Due to the diversity of data used, processing and testing conditions are accounted for in the universal knock down factors. Although only biaxial test data was used in this study, one could use any type of laminate test data to augment the population of Carbon/Epoxy data used for determination of universal knock down factors.

4. REFERENCES

1. E. J. Barbero and J. M. Gutierrez, “Determination of basis values from experimental data for fabrics and composites” In *SAMPE 2012 Conference and Exhibition*, Baltimore, May 21-24, 2012.
2. E. J. Barbero, “Universal carpet plots for stiffness and strength design” In *CAMX 2017*, Orlando, FL, December 11-14, volume TP17 (0006). SAMPE, 2017.
3. E. J. Barbero *Introduction to Composite Materials Design*, CRC Press, Boca Raton, FL, third edition, 2018.
4. NIST/SEMATECH, *e-handbook of statistical methods*, NIST, 2013. urlhttp://www.itl.nist.gov/div898/handbook/eda/section3/probplot.htm.

5. J. Tomblin, J. Sherraden, W. Seneviratne, and K. S. Raju, "A-basis and B-basis design allowables for epoxy-based prepreg Toray T700gc-12k-31e/2510 unidirectional tape" *AGATE-WP3.3-033051-132*, 2002.
6. Composite Materials Handbook, volume 2. *Polymer Matrix Composites: Material Properties,* SAE International, 2012.