POST-BUCKLING AND FIRST-PLY FAILURE OF THIN-WALLED FRAMES AND COLUMNS MADE OF COMPOSITE MATERIALS

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ABSTRACT

First-ply failure (FPF) of composite thin-walled columns and frames is investigated in the buckled structure. Elastic behavior is assumed during the pre-buckling and buckling states, and along part of the post-buckling equilibrium path. The Tsai-Hill criterion is employed to identify failure of one ply at one specific location. The technique of analysis is based on evaluation of the post-buckling elastic path, and using FPF as a constraint in the analysis. The structure is modeled using a few degrees of freedom to include local and/or overall buckling modes. The theoretical results are compared with experimental bounds and show good agreement for local buckling of isolated columns. There are no experimental results for frames in the literature, but theoretical results are presented for a simple case with asymmetric bifurcation.

KEYWORDS

Buckling, Composites materials, Frames, Ply-failure, Post-buckling, Thin-Walled Columns.

INTRODUCTION

This paper is about the post-buckling behavior of columns and frames made with composite materials. The members of the structures considered are formed by pultrussion using high-strength e-glass fibers embedded in a polymer resin. Because of limitations of cost and maximum thickness, all pultruded structural shapes have thin walls and are thus liable to buckling under compression. Most studies on the buckling of composite columns are limited to the critical load itself. Beyond elastic buckling the structure may have a stable post-buckling path and further load can be taken with large deflections. How far is the structure from a maximum load is not given by such eigenvalue calculations. Some mode of failure criterion is needed to compute more realistic maximum loads and thus guide the design and improve material performance.

The structural shapes considered in this paper are wide flange *I*-sections. A number of researchers have addressed the problem of post-buckling behavior of *I*-columns, but consideration of frames made with such shapes is not easily found in the literature, although they are frequently employed in practice.

This paper deals with local and global buckling modes of thin-walled members and frames, constructed using fiber-reinforced polymers. Such members are fabricated by pultrussion and are assembled to build frame structures for industrial purposes. The paper includes theoretical models based on simple analytical and numerical procedures to account for buckling and post-buckling. The formulation uses the total potential energy and the theory of elastic stability, so that an asymptotic solution of the post-buckling path is obtained. Rather than computing the non-linear material behavior along the path, the present approach is to include first-ply failure (FPF) as a material constraint to the elastic buckling process. To achieve that, first ply failure is modeled using Tsai-Hill criterion, so that it is possible to identify the layer inside the laminate which fails for the first time. A safety factor is computed with respect to a FPF criterion, and thus the weakness in the strength characteristics are visualized. A direction of improvement in the material is then possible.

The present work is based on preliminary findings by Almanzar & Godoy (1997b) on columns, and the results are compared in Section 3 with experiments. The analysis is extended in Section 4 to study some frame behavior.

FORMULATION OF THE PROBLEM

The investigation of post-buckling and FPF leads to a non-linear problem. Advanced computer models that include geometrically non-linear behavior and non-linear material properties are available to the engineer; however, even for the present day computing facilities this is an expensive computation, and more so if different possibilities need to be considered. A more convenient solution can be obtained by coupling the general theory of elastic stability with a FPF condition of the composite. The basic approach is to evaluate the post-buckling response of the structure using elastic properties for the material, and including the non-linear material behavior as a constraint to the post-critical path. Such approach has been discussed by Hutchinson (1974), Croll (1983) and others. All previous studies have focused on metal structures and the von-Mises criterion, while in the present work the formulation and results are extended to laminated composites. There are several ways to include the material behavior as a constraint, as pointed out by Godoy (1998).

Because of limitations of space, only a description of the formulation is presented here. The first step in the analysis is the computation of equilibrium paths using an elastic constitutive relation. The total potential energy of the system is built using a displacement field in terms of a small number of degrees of freedom. For the analysis of an isolated column, local and global buckling modes can be modeled as in Godoy, Barbero & Raftoyiannis (1995). The displacements are assumed for the web and the flanges, and compatibility conditions are employed to put them together as part of the more complex shape. Classical lamination theory is used to compute the stiffness of web and flanges. Such formulation for simply supported columns has also been used by Almanzar & Godoy (1997a) for sensitivity analysis of the post-buckling path.

Only global buckling within each member is considered for frames, although a buckling mode can have displacements in all members. Each member is assumed to have displacements in the form of cubic polynomials. The constitutive equations are those of the laminated composite for the flanges and for the web, as in the case of the isolated column. The energy contribution of each element of the frame is assembled into the total potential energy V of the structure. Notice that because non-linearity of the membrane strain is included, then V becomes quartic in the degrees of freedom of the system.

The energy V is next used in the context of the general theory of elastic stability to compute the fundamental equilibrium path, the critical state, and the post-critical equilibrium path. Further details of the energy formulation can be seen in Flores & Godoy (1992), and Almanzar (1998). A main difference between the behavior of an isolated column and of a frame is that bifurcation is symmetric in the former, but it can be symmetric or asymmetric in the latter.

The second step in the analysis is the implementation of a FPF criterion that can be used together with the stability and post-buckling formulations. In this work we have used the Tsai-Hill quadratic criterion (see Tsai, 1988) which can be written as

$$(\sigma_1/S_L)^2 - (\sigma_1\sigma_2/S^2_L) + (\sigma_2/S_T)^2 + (\tau_{12}/S_{LT})^2 = 1 \tag{1}$$

where $(\sigma_1, \sigma_2, \tau_{12})$ are the stresses in the ply considered; S_L is the longitudinal compressive strength; S_T is the transverse compressive strength; and S_{LT} is the longitudinal shear strength. Evaluation of the above strengths of a unidirectional or fabric ply is determined from relatively simple tests. In order to avoid FPF the left-hand side of Eq. (1) should be <1, and failure is predicted if it is ≥ 1. The failure surface generated by this equation is an ellipsoid in the $(\sigma_1, \sigma_2, \tau_{12})$ space, which is symmetric about the origin if equal strengths are assumed in tension and in compression. In most composites the tensile and the compressive strengths are different.

In the application of the failure criterion the next step is the computation of the stresses in a lamina from the displacements along the equilibrium path. The stresses should also be computed at several locations in the column because one does not know a priori where FPF occurs. These locations are to be searched along the length of each member, with the height of the web, and along the flanges. Only a few points are employed but care should be taken to include the location of more severe stresses. Thus, the stresses in each lamina are computed and compared with the Tsai-Hill criterion.

Because the formulation first constructs the energy and then employs its derivatives, a symbolic manipulator is very convenient. The above formulation has been implemented in a computer code written in the symbolic manipulator MAPLE V.

FIRST-PLY FAILURE OF COLUMNS IN EXPERIMENTS AND IN THE MODEL

Numerical as well as experimental work in composite columns and frames indicate that the material remains elastic until linear buckling occurs. For post-buckling equilibrium states, however, there is evidence of damage associated to the large post-buckling displacements.

To investigate damage, several columns were tested by Barbero and co-workers at West Virginia University, as reported in Dede (1996) and Barbero & Trovillon (1998). Tests were performed on a column with dimensions $b = d = 304.8mm$, thickness $t = 12,7mm$ $(12\times12\times1/2in)$, $A = 112.9mm^2$, $I = 5998.5mm_4$, and $L = 1816mm$. The sections tested contained roving (unidirectional fibers) and continuous strand mat, with axial stiffness $EA = 289MN$ and bending stiffness $EI = 1.5MNm^2$.

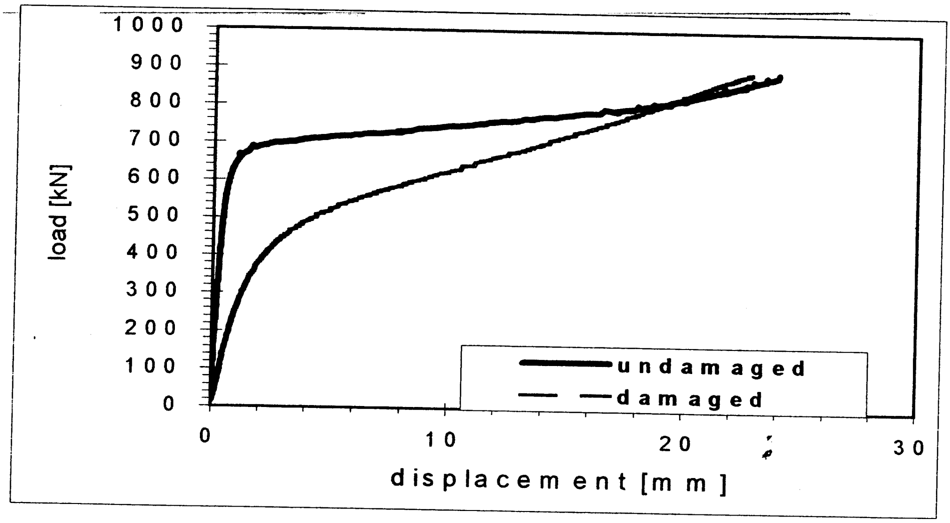


Figure 1

The sample was loaded using an MTS hydraulic machine at a loading rate of *89kN/min*. The ends of the column were simple supported. Load and displacements of the flanges were recorded using a data acquisition system. Post-buckling displacements of approximately *20mm* were recorded during first loading of the column up to *900kN* (approximately 30% higher than the critical load), and are shown as a solid curve in Figure 1. The structure had damage in the material at this stage. This is next observed by unloading and reloading, in which case the equilibrium path is the dotted line in Figure 1. Clearly, the initial stiffness of the structure has now changed, with a decrease due to damage. In the undamaged structure one may observe a bifurcation at a load of approximately 700kN; the damaged column, on the other hand, is dominated by the non-linear response.

The present global experimental measurements do not allow to detect the occurrence of FPF; however, it gives bounds within which theoretical predictions of FPF should be. The column has been studied using the theoretical and numerical models described in Section 2. Buckling was computed for three half waves in the longitudinal direction, completely in agreement with experimental tests. First-ply failure was detected at a load approximately 15% higher than the critical load, thus within the bounds found experimentally. A difference between the present model and the experiments is the mode shape: there is a mode attenuation in the experiments, while this is not taken into account by the present simplified model.

FIRST-PLY FAILURE IN BUCKLED FRAME STRUCTURES

Next, a simple frame structure is considered, as shown in Figure 2a. This is a four-bar frame, but because sway buckling is prevented, then the analysis can be performed with only two members and conditions of symmetry. Each member of the frame is a thin-walled *I*-section, with dimensions *6x6x1/4in* and *L=200in (5m)*, constructed with 7 layers of thickness *0.0357in* each one. The internal angles between the vertical and inclined members are 11.25°, and *L1=5m*. Fiber volume fraction is 0.3, and for e-glass we employed $E_1=E_2=10.6\times 10^6 psi$ and $\nu=0.22$, while for epoxi the properties are $E_1=E_2=0.5\times 10^6 psi$ and $\nu=0.35$.

For simplicity of the presentation only results for one element for each bar are reported here, with the active degrees of freedom shown in Figure 2b. For this problem, the fundamental equilibrium path has vertical displacements Q_2^F and Q_5^F. The critical state is a linear eigenvalue problem leading to a critical load factor $\Lambda^C=30,998.2lb$.

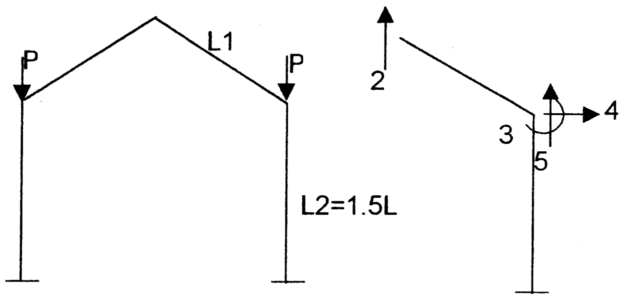


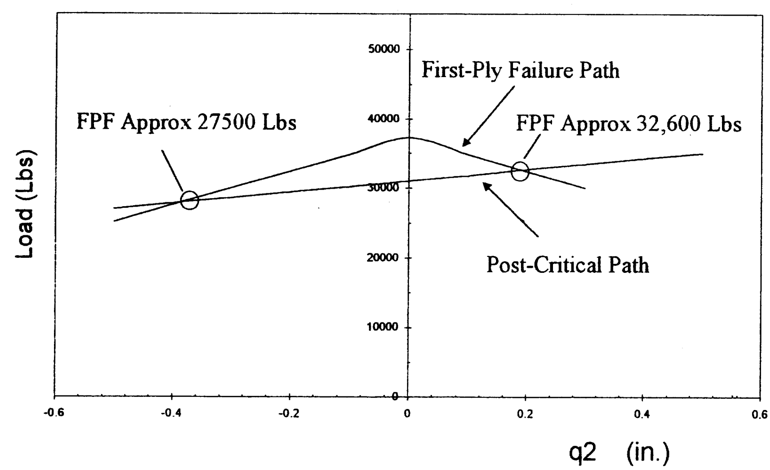
Figure 2

For the post-buckling equilibrium path the perturbation parameter s chosen is the vertical displacement at the apex of the frame, $s=Q_2$. The post-buckling path is written using the regular perturbation expansion

$$\Lambda(s) = \Lambda^C + s\,\Lambda^{(1)} + s^2\,\Lambda^{(2)} + \ldots \qquad (2)$$

For the evaluation of the slope $\Lambda^{(1)}$ and curvature $\Lambda^{(2)}$ of the post-buckling path we require the coefficients A, B, C (see for example Flores & Godoy, 1992), which in this case result in $C= -395$, $B=$

Figure 3



-0.0245, $A=-0.152\times10^{-5}$. Because $C\neq0$, then we are in the presence of an asymmetric bifurcation. The slope of the post-buckling path results in $\Lambda^{(1)} = 1/2\ C/B = 8,064$ and $\Lambda^{(2)} = 40,320$. One can now compute the post-buckling path of the structure and this has been plotted in Figure 3.

A FPF path has to be computed next for each location selected in the structure. The Tsai-Hill criterion, Eq. (1), is used with the material parameters already used for the isolated column. From the displacement field we have computed the stresses and the load-path, which is also indicated in Figure 3. On the stable part of the path, FPF occurs at a load which is 5% higher than the critical load, with displacements of 0.2in; while on the unstable branch, FPF is predicted for 88% of the critical load with displacements of 0.4in.

On the unstable branch the influence of imperfections is to reduce the maximum load in the elastic range; however, because the post-buckling path is relatively shallow in most frame structures, then it is expected that the present approximation should be a good estimate of the post-buckling FPF.

CONCLUSIONS

The theoretical approach followed in this work is aimed at the evaluation of FPF in laminated composite for buckled structures. The theme structures considered are thin-walled columns and frames assembled with such members. The specific materials studied are fiber reinforced polymers using e-glass and formed by pultrussion. The elastic post-buckling behavior is computed using the general theory of elastic stability, from which a perturbation expansion of the post-buckling path is obtained. FPF is introduced as a material constraint, and the FPF path is evaluated together with the post-buckling equilibrium path. FPF occurs at the intersection of the two paths.

For isolated columns, the paper considers the local buckling modes, and results are compared with bounds of experimental results. In the experiments it is not possible to identify FPF; however, the results for the columns with damage represent upper bounds to the theoretical results.

The displacements w_{FPF} for FPF normalized with respect to the thickness t of the walls are of $0.5 \geq w_{FPF}/t \geq 2$. This occurs for cases as diverse as column local buckling or frame buckling in global mode. This means that soon after buckling the structure reaches FPF. It is not necessary to advance into too large displacements to have failure in one ply.

In its present form, the model presented can be used as a limit to elastic behavior. Damage of the structure occurs following FPF and this was shown to change the initial stiffness for subsequent unloading and reloading of the structure. Such important changes are undesirable, so that care must be taken that the structure (either column or frame) works below a FPF limit.

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