Calculation of Prony Series Parameters From Dynamic Frequency Data

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1 Definition of a Prony Series

Let $G_R(t)$ be the shear stress relaxation modulus. Define G_∞ and G_0 by the following limits:

$$G_\infty = \lim_{t\to\infty} G_R(t) \tag{1}$$

$$G_0 = G_R(0). \tag{2}$$

From the shear relaxation modulus we can define a dimensionless relaxation modulus from:

$$g_R(t) = \frac{G_R(t)}{G_0}. \tag{3}$$

The normalized shear stress relaxation modulus is often represented by a series expansion in exponential terms:

$$g_R(t) = 1 - \sum_{i=1}^{N} g_i \left[1 - e^{-t/\tau_i}\right], \tag{4}$$

where g_i and τ_i are material parameters. This series expansion is called the *Prony series*.

2 Dynamic Frequency Experiments

The viscoelastic behavior of a material is often determined from dynamic vibrational experiments. In these experiments the material is exposed to small strain vibrations and the resulting storage modulus (G') and loss modulus (G'') are determined as a function of the applied frequency: $G'(\omega)$, $G''(\omega)$.

3 Conversion from Dynamic Frequency Data to Prony Series Data

In most finite element packages the linear viscoelasticity model is specified by a Prony series (Eq. 4). If only dynamic frequency data is available for a material then the Prony series can be determined from the following implicit equations [see the ABAQUS theory manual for a derivation]:

$$G'(\omega) = G_0 \left[1 - \sum_{i=1}^{N} g_i\right] + G_0 \sum_{i=1}^{N} \frac{g_i \tau_i^2 \omega^2}{1 + \tau_i^2 \omega^2} \tag{5}$$

$$G''(\omega) = G_0 \sum_{i=1}^{N} \frac{g_i \tau_i \omega}{1 + \tau_i^2 \omega^2}. \tag{6}$$

These equations determine the storage modulus and the loss modulus for a given Prony series. In this case we are interested in the reverse relationship, that is, if we know $G'(\omega)$ and $G''(\omega)$ then what is the Prony series (G_0, N, g_i, τ_i)?

To solve this problem we can use the following approach.

1. Pick a value for N.
2. Guess values for G_0, g_i, τ_i.
3. Calculate $G'(\omega)$ and $G''(\omega)$.
4. Calculate the residual between the calculated dynamic data and the experimental dynamic data.
5. Optimize the parameters G_0, g_i, and τ_i using any minimization algorithm (e.g. the Simplex method). Goto 3.

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